Operations on Continuous-Time Signals

David W. Graham
EE 327
Continuous-Time Signals

• Continuous-Time Signals
  – Time is a continuous variable
  – The signal itself need not be continuous

• We will look at several common continuous-time signals and also operations that may be performed on them
Unit Step Function $\rightarrow u(t)$

$u(t) = \begin{cases} 
1 & t \geq 0 \\
0 & t < 0 
\end{cases}$

- Used to characterize systems
- We will use $u(t)$ to illustrate the properties of continuous-time signals
## Operations of CT Signals

1. **Time Reversal**  
   
   \[ y(t) = x(-t) \]

2. **Time Shifting**  
   
   \[ y(t) = x(t-t_d) \]

3. **Amplitude Scaling**  
   
   \[ y(t) = Bx(t) \]

4. **Addition**  
   
   \[ y(t) = x_1(t) + x_2(t) \]

5. **Multiplication**  
   
   \[ y(t) = x_1(t)x_2(t) \]

6. **Time Scaling**  
   
   \[ y(t) = x(at) \]
1. Time Reversal

• Flips the signal about the y axis
• \( y(t) = x(-t) \)

ex. Let \( x(t) = u(t) \), and perform time reversal

**Solution**: Find \( y(t) = u(-t) \)

Let “\( a \)” be the argument of the step function \( \rightarrow u(a) \)

\[
u(a) = \begin{cases} 
1 & a \geq 0 \\
0 & a < 0 
\end{cases}
\]

Let \( a = -t \), and plug in this value of “\( a \)”

\[
u(-t) = \begin{cases} 
1 & t \leq 0 \\
0 & t > 0 
\end{cases}
\]
2. Time Shifting / Delay

- $y(t) = x(t - t_d)$
- Shifts the signal left or right
- Shifts the origin of the signal to $t_d$

- Rule $\rightarrow$ Set $(t - t_d) = 0$ (set the argument equal to zero)
  $\rightarrow$ Then move the origin of $x(t)$ to $t_d$

- Effectively, $y(t)$ equals what $x(t)$ was $t_d$ seconds ago
2. Time Shifting / Delay

ex. Sketch \( y(t) = u(t - 2) \)

\[
y(a) = \begin{cases} 
1 & a \geq 0 \\
0 & a < 0 
\end{cases} = \begin{cases} 
1 & t - 2 \geq 0 \\
0 & t - 2 < 0 
\end{cases} = \begin{cases} 
1 & t \geq 2 \\
0 & t < 2 
\end{cases}
\]

Method 2 (by inspection)
Simply shift the origin to \( t_d = 2 \)
3. Amplitude Scaling

- Multiply the entire signal by a constant value
- \( y(t) = Bx(t) \)

ex. Sketch \( y(t) = 5u(t) \)
4. Addition of Signals

- Point-by-point addition of multiple signals
- Move from left to right (or vice versa), and add the value of each signal together to achieve the final signal
- \( y(t) = x_1(t) + x_2(t) \)

- Graphical solution
  - Plot each individual portion of the signal (break into parts)
  - Add the signals point by point
4. Addition of Signals

ex. Sketch \( y(t) = u(t) - u(t - 2) \)

First, plot each of the portions of this signal separately

- \( x_1(t) = u(t) \) \( \rightarrow \) Simply a step signal
- \( x_2(t) = -u(t-2) \) \( \rightarrow \) Delayed step signal, multiplied by -1

Then, move from one side to the other, and add their instantaneous values
5. Multiplication of Signals

• Point-by-point multiplication of the values of each signal

  \[ y(t) = x_1(t)x_2(t) \]

• Graphical solution
  – Plot each individual portion of the signal (break into parts)
  – Multiply the signals point by point
5. Multiplication of Signals

ex. Sketch $y(t) = u(t) \cdot u(t - 2)$

First, plot each of the portions of this signal separately

- $x_1(t) = u(t)$ → Simply a step signal
- $x_2(t) = u(t-2)$ → Delayed step signal

Then, move from one side to the other, and multiply instantaneous values
6. Time Scaling

- Speed up or slow down a signal
- Multiply the time in the argument by a constant

\[ y(t) = x(at) \]

- \(|a| > 1 \) → Speed up \( x(t) \) by a factor of “a”
- \(|a| < 1 \) → Slow down \( x(t) \) by a factor of “a”

- Key → Replace all instances of “t” with “at”
ex. Let \( x(t) = u(t) - u(t - 2) \)

Sketch \( y(t) = x(2t) \)

Replace all \( t \)'s with \( 2t \)

\[ y(t) = x(2t) = u(2t) - u(2t - 2) \]

- Turns on at \( 2t \geq 0 \) \( t \geq 0 \)
- No change
- Turns on at \( 2t - 2 \geq 0 \) \( t \geq 1 \)

This has effectively “sped up” \( x(t) \) by a factor of 2

(What occurred at \( t=2 \) now occurs at \( t=2/2=1 \))
6. Time Scaling

ex. Let \( x(t) = u(t) - u(t - 2) \)
Sketch \( y(t) = x(t/2) \)

Replace all \( t \)'s with \( t/2 \)

\[
y(t) = x(t/2) = u(t/2) - u((t/2) - 2)
\]

Turns on at \( t/2 \geq 0 \)
\( t \geq 0 \)
No change

Turns on at \( t/2 - 2 \geq 0 \)
\( t \geq 4 \)

This has effectively “slowed down” \( x(t) \) by a factor of 2
(What occurred at \( t=1 \) now occurs at \( t=2 \))
Combinations of Operations

• Combinations of operations on signals
  – Easier to Determine the final signal in stages
  – Create intermediary signals in which one operation is performed
**ex. Time Scale and Time Shift**

Let \( x(t) = u(t + 2) - u(t - 4) \)

Sketch \( y(t) = x(2t - 2) \)

Can perform either operation first

Method 1 \( \rightarrow \) Shift then scale

Let \( v(t) = x(t - b) \) \( \rightarrow \) Time shifted version of \( x(t) \)

Then \( y(t) = v(at) = x(at - b) \)

Replace "t" with the argument of "v"

Match up "a" and "b" to what is given in the problem statement

\[
\begin{align*}
\text{at} - b &= 2t - 2 \\
\text{(Match powers of t)}
\end{align*}
\]

Therefore, shift by 2, then scale by 2
ex. Time Scale and Time Shift

Let \( x(t) = u(t + 2) - u(t - 4) \)
Sketch \( y(t) = x(2t - 2) \)

Can perform either operation first

Method 2 \( \rightarrow \) Scale then shift

Let \( v(t) = x(at) \) \( \rightarrow \) Time scaled version of \( x(t) \)
Then \( y(t) = v(t - b) = x(a(t - b)) = x(at - ab) \)
Replace “t” with the argument of “v”
Match up “a” and “b” to what is given in the problem statement

\[ at - ab = 2t - 2 \]
(Match powers of t)
\[ a = 2 \]
\[ ab = 2, \ b = 1 \]

Therefore, scale by 2, then shift by 1
ex. Time Scale and Time Shift

• Note – The results are the same
• Note – The value of b in Method 2 is a scaled version of the time delay
  – $t_d = 2$
  – Time scale factor = 2
  – New scale factor = $2/2 = 1$