Discrete-Time Convolution

EE 327

Addition Method of Discrete-Time Convolution

- Produces the same output as the graphical method
- Effectively a “short cut” method

Let x[n] = 0 for all n<N (sample value N is the first non-zero value of x[n]
Let h[n] = 0 for all n<M (sample value M is the first non-zero value of h[n]

$$\therefore y[n] = x[n] * h[n] = \begin{cases} 0 & for \quad n < M + N \\ \sum_{i=N}^{M} x[i]h[n-i] & for \quad n \geq M + N \end{cases}$$

To compute the convolution, use the following array

Discrete-Time Convolution Array

x[N]	x[N+1]	x[N+2]	x[N+3]	…	1st row – values of x[n]
h[M]	h[M+1]	h[M+2]	h[M+3]	…	2nd row – values of h[n]
x[N]h[M]	x[N+1]h[M]	x[N+2]h[M]	x[N+3]h[M]	…	(1st row) x (1st element of 2nd row)
	x[N]h[M+1]	x[N+1]h[M+1]	x[N_2]h[M+1]	…	(1st row) x (2nd element of 2nd row)
		x[N]h[M+2]	x[N+1]h[M+2]	…	(1st row) x (3rd element of 2nd row)
+			⋮		
y[N+M]	y[N+M+1]	y[N+M+2]	y[N+M+3]	…	

Sum down a column

The resulting values in the output sequence

Discrete-Time Convolution Example

Find the output of a system if the input and impulse response are given as follows.

$$x[n] = \delta[n+1] + 2\delta[n] + 3\delta[n-1] + 4\delta[n-2]$$
$$h[n] = -\delta[n+2] + 5\delta[n+1] + 3\delta[n]$$

Solution

Then, N = -1 → Index of the first non-zero value of x[n]

M = -2 → Index of the first non-zero value of h[n]

Next, write an array

Discrete-Time Convolution Example

1	2	3	4			Coefficients of x[n]
-1	5	3				Coefficients of h[n]
-1	-2	-3	-4			First Row times (-1)
	5	10	15	20		First Row times (5)
		3	6	9	12	First Row times (3)
-1	3	10	17	29	12	Summation of columns

y[n] = 0 for n < N+M = -3

$$y[n] = -\delta[n+3] + 3\delta[n+2] + 10\delta[n+1] + 17\delta[n] + 29\delta[n-1] + 12\delta[n-2]$$

Numerical Convolution Using MATLAB

- Define x[n] and h[n]
  - `xx = [5 7 -9 10];`
  - `hh = [1 2 3];`
  - May also be useful to use the `zeros` and `ones` functions (especially for matching up sample values)
- Use the `conv` function
  - `yy = conv(xx,hh);`