Numerical Convolution

EE 327

Numerical Convolution

- Numerical evaluation of the convolution integral
- For difficult-to-solve convolution problems
- We will focus on the Matlab `conv` function

Discretizing the Time

Convolution Integral

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$$

Discretize the Time

$t = nT$

n → sample value

T → sampling period

$$y(nT) = \int_{-\infty}^{\infty} h(\lambda)x(nT-\lambda)d\lambda$$

Break the Integral into Pieces (T-second intervals)

$$y(nT) = \cdots + \int_{0}^{T} h(\lambda)x(nT-\lambda)d\lambda + \int_{T}^{2T} h(\lambda)x(nT-\lambda)d\lambda$$

$$+ \int_{iT}^{iT+T} h(\lambda)x(nT-\lambda)d\lambda + \cdots$$

$$= \sum_{i=-\infty}^{\infty} \int_{iT}^{iT+T} h(\lambda)x(nT-\lambda)d\lambda$$

Let T be sufficiently small that $h(\lambda)x(nT-\lambda)$ is relatively constant over T

Then

$$\left.\begin{aligned} h(\lambda) &= h(iT) \\ x(nT-\lambda) &= x(nT-iT) \end{aligned}\right\} \text{ for } iT \le \lambda < iT+T$$

$$\begin{aligned} y(nT) &= \sum_{i=-\infty}^{\infty} \int_{iT}^{iT+T} h(iT)x(nT-iT)\,d\lambda \\ &= \sum_{i=-\infty}^{\infty} h(iT)x(nT-iT) \int_{iT}^{iT+T} d\lambda \\ &= \sum_{i=-\infty}^{\infty} h(iT)x(nT-iT)(iT+T-iT) \\ &= \sum_{i=-\infty}^{\infty} T\cdot h(iT)x(nT-iT) \end{aligned}$$

$$y(nT) = \sum_{i=-\infty}^{\infty} T \cdot h(iT)x(nT - iT)$$

In discrete-time notation

$$y[n] = \sum_{i=-\infty}^{\infty} T \cdot h[i]x[n-i]$$

Numerical Convolution in Matlab

- Use `conv` function
- `yy = conv(T*hh,xx);`
- `yy = conv(hh,T*xx);`

Keys to Numerical Convolution

- Convert to discrete time
- The smaller the sampling period, T, the more exact the solution
- Tradeoff → computation time vs. exactness of solution
- Remember to account for T in the convolution → ex. T*hh