State-Space Modeling

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State-Space Modeling

- Alternative method of modeling a system than
  - Differential / difference equations
  - Transfer functions
- Uses matrices and vectors to represent the system parameters and variables

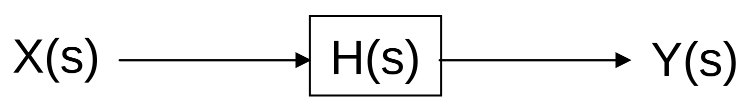
Motivation for State-Space Modeling

- Easier for computers to perform matrix algebra
  - e.g. MATLAB does all computations as matrix math
- Handles multiple inputs and outputs
- Provides more information about the system
  - Provides knowledge of internal variables (states)

$\Rightarrow$Primarily used in complicated, large-scale systems

Transfer Functions vs. State-Space Models

- Transfer functions provide only input and output behavior
  - No knowledge of the inner workings of the system
  - System is essentially a “black box” that performs some functions



- State-space models also represent the internal behavior of the system

Definitions

V – Input vector

- Can be multiple inputs
- Written as a column vector

$$v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_R(t) \end{bmatrix}$$

Y – Output vector

- A function of the input and the present state of the internal variables

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_M(t) \end{bmatrix}$$

Definitions

X – State vector

- Information of the current condition of the internal variables
- N is the "dimension" of the state model (number of internal state variables)

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix}$$

$\dot{X}$ – "Next state" vector

- Derivative of the state vector
- Provides knowledge of where the states are going
  - Direction (+ or -)
  - How fast (magnitude)
- A function fo the input and the present state of the internal variables

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_N(t) \end{bmatrix}$$

State-Space Equations

General form of the state-space model

Two equations – $\dot{x}(t)$
$y(t)$

$$\dot{x}(t) = f\left(x(t), v(t), t\right)$$
$$y(t) = g\left(x(t), v(t), t\right)$$

Linear State-Space Equations

$$\dot{x}(t) = Ax(t) + Bv(t)$$
$$y(t) = Cx(t) + Dv(t)$$

$x(t),\ \dot{x}(t) \rightarrow N \times 1$ vectors

$v(t) \rightarrow R \times 1$ vector

$y(t) \rightarrow M \times 1$ vector

$A \rightarrow N \times N$ system matrix

$B \rightarrow N \times R$ input matrix

$C \rightarrow M \times N$ output matrix

$D \rightarrow M \times R$ matrix representing direct coupling from system inputs to system outputs

If A, B, C, D are constant over time, then the system is also time invariant
$\rightarrow$ Linear Time Invariant (LTI) system

Construction of State Equations from a Differential Equation

(Let there be no derivatives of the input)

- The dimension of the state equations (number of state variables) should equal the order of the differential equation
- Let one state variable equal the output (y(t))
- Let one state variable equal the derivative of the output

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- Let one state variable equal the (N-1)-th derivative of the output (where N is the order of the differential equation)
- Find the derivative of each of the newly defined state equations
  - In terms of the other state variables and the outputs
- Write the state equations