Efficient GM estimation of a Cliff and Ord panel data model with random effects*

Gianfranco Piras†

December 18, 2012

Abstract

The present paper suggests an estimation procedure for a Cliff and Ord type spatial panel data model with random effects. Building on existing literature, the paper suggests an estimation procedure that *i*) considers all the moment conditions in Kapoor *et al.* (2007) and *ii*) allows for the presence of explanatory variables that do not vary over time. Our Monte Carlo results demonstrate that the estimation procedure proposed in this paper is very effective.

Keywords: Spatial panel data models, Instrumental Variables, Random Effects estimator

*Previous versions of the paper were presented at the 5^{th} world conference of the Spatial Econometrics Association, at the 51^{st} Annual Meeting of the Southern Regional Science Association, at the 51^{st} Annual Meeting of the Western Regional Science Association, and at a seminar at the Richmond Federal Reserve Bank. Comments from all the discussants and participants were greatly appreciated. The paper also benefited from the comments and suggestions of the guest editors of this special issue, and two referees. Last but not least, the paper is dedicated to Harry Kelejian.

†Regional Research Institute, West Virginia University, 886 Chestnut Ridge Road, Morgantown, WV 26506-6825. email: Gianfranco.Piras@email.wvu.edu

1 Introduction

In the last few years, there has been an increasing interest in the theoretical, as well as the applied literature on spatial panel data models.[1] Kapoor *et al.* (2007) consider a spatial panel data model with random effects. Mutl & Pfaffermayr (2011) have extended the estimation procedure to a Cliff and Ord type model including the spatial lag of the dependent variable as well as a spatially lagged one-way error component model. They implement instrumental variables estimation under both the fixed and the random effects specifications. However, in establishing their estimation procedure, they do not use all moment conditions derived in Kapoor *et al.* (2007) for the random effects specification. Additionally, as will become clear later, their procedure would produce estimates of σ_1^2 that are biased and inefficient when the random effects model includes explanatory variables that are constant over time.[2] In the present paper, an estimation procedure is suggested for a random effects panel data model that *i*) considers all the moment conditions in Kapoor *et al.* (2007) and *ii*) allows for the presence of explanatory variables that do not vary over time. The multi-step estimation procedure proposed in this paper is similar in spirit to the one presented in Fingleton (2008) for a model with spatial moving average errors. To carry out the IV estimation, Fingleton (2008) uses as instruments a linear independent subset of the exogenous variables. This paper follows more closely the fixed and between effects two stage least squares estimator proposed by Baltagi & Liu (2011) to consistently estimate the within and between residuals. The Monte Carlo results demonstrate that the procedure is effective in small samples.

The model is specified in Section 2, while the suggested estimation procedure is laid out in Section 3. Section 4 describes the design of the Monte Carlo analysis and discusses the main evidence. A final section concludes the paper.

[1]Recent contributions include, among others, Anselin *et al.* (2008), Kapoor *et al.* (2007), Baltagi *et al.* (2007c, 2003), Baltagi & Liu (2008), Baltagi *et al.* (2007b), Debarsy & Ertur (2010), Elhorst (2003), Elhorst & Freret (2009), Elhorst (2008, 2009, 2010), Elhorst *et al.* (2010), Lee & Yu (2010a,b,c), Mutl (2006), Mutl & Pfaffermayr (2011), Pesaran & Tosetti (2011), Yu & Lee (2010), Yu *et al.* (2008), Parent & LeSage (2010)

[2]A typical example of this is when the researcher includes a distance variable. However, since Mutl & Pfaffermayr (2011) derive a spatial Hausman test, this cannot really be considered a limitation of their paper. Their estimation procedure present no problem in the fixed effects case.

2 The model

This paper considers a general static panel model that includes a spatial lag of the dependent variable and spatial autoregressive disturbances:[3]

$$y = \lambda(I_T \otimes W)y + X\beta + u \tag{1}$$

where y is an $nT \times 1$ vector of observations on the dependent variable; X is an $nT \times k$ matrix of observations on the non-stochastic exogenous regressors;[4] I_T an identity matrix of dimension T; W is the $n \times n$ spatial weighting matrix of known constants whose diagonal elements are set to zero; and λ the corresponding spatial parameter.[5] The observations are ordered first by time and then by individual units. The model can be rewritten more compactly as

$$y = Z\delta + u \tag{2}$$

where $Z = [(I_T \otimes W)y, X]$, and $\delta = [\lambda, \beta']'$. The disturbance term follows a first order spatial autoregressive process of the form:

$$u = \rho(I_T \otimes W)u + \varepsilon \tag{3}$$

where W is the spatial weighting matrix and ρ the corresponding spatial autoregressive parameter.[6] To further allow for the innovations to be correlated over time, the innovations vector in (3) is assumed to follow an error component structure

$$\varepsilon = (\iota_T \otimes I_n)\mu + \nu \tag{4}$$

where μ is the vector of cross-sectional specific effects that is assumed to be $IID \sim (0, \sigma^2_\mu I_n)$; ν is a vector of innovations that vary both over cross-

[3]As long as instruments are available, the model could easily be extended to the presence of additional (other than the spatial lag) endogenous variables (see, e.g. Fingleton & Le Gallo 2008).

[4]The regressors matrix is assumed to be of full column rank and its elements are assumed to be asymptotically bounded in absolute value.

[5]The spatial weighting matrix is assumed to be row-normalized. Also, the row and column sums of the spatial weighting matrix are uniformly bounded in absolute value. In addition, both $|\rho| < 1$ and $|\lambda| < 1$ and the row and column sums of $(I_n - \lambda W)^{-1}$ and $(I_n - \rho W)^{-1}$ are also uniformly bounded in absolute value.

[6]To simplify notation, the spatial weighting matrix in the error term and the one that multiplies the dependent variable in equation (1) are assumed to be the same. However, this does not need to be the case in general.

sectional units and time periods and is assumed to be $IID \sim (0, \sigma_\nu^2 I_{nT})$; and ι_T is a vector of ones of dimension T. Also, ν and μ are independent of each other and the regressors matrix. One can rewrite (3) as

$$u = [I_T \otimes (I_n - \rho W)^{-1}]\varepsilon \tag{5}$$

It follows that the variance-covariance matrix of u is

$$\Omega_u = [I_T \otimes (I_n - \rho W)^{-1}]\Omega_\varepsilon [I_T \otimes (I_n - \rho W')^{-1}] \tag{6}$$

where $\Omega_\varepsilon = \sigma_\nu^2 Q_0 + \sigma_1^2 Q_1$, with $\sigma_1^2 = \sigma_\nu^2 + T\sigma_\mu^2$, $Q_0 = \left(I_T - \frac{J_T}{T}\right) \otimes I_n$, $Q_1 = \frac{J_T}{T} \otimes I_n$ and $J_T = \iota_T \iota_T'$, is the typical variance-covariance matrix of a one-way error component model.

Furthermore, it should be noted that

$$\begin{aligned} E[(I_T \otimes W)yu'] =& E[(I_T \otimes W)[I_T \otimes (I_n - \lambda W)](X\beta + u)u'] \\ =& (I_T \otimes W)[I_T \otimes (I_n - \lambda W)]\Omega_u \neq 0 \end{aligned} \tag{7}$$

and, therefore, OLS will be inconsistent.

3 Estimation issues

Kapoor *et al.* (2007) suggest a generalization of the generalized moment estimator introduced by Kelejian & Prucha (1999) for estimating the spatial autoregressive parameter (ρ) and the two variance components of the disturbance process (σ_1^2 and σ_ν^2). Specifically, they define three sets of GM estimators based on the following moment conditions:

$$E \begin{bmatrix} \frac{1}{n(T-1)}\varepsilon' Q_0 \varepsilon \\ \frac{1}{n(T-1)}\bar{\varepsilon}' Q_0 \bar{\varepsilon} \\ \frac{1}{n(T-1)}\bar{\varepsilon}' Q_0 \varepsilon \\ \frac{1}{n}\varepsilon' Q_1 \varepsilon \\ \frac{1}{n}\bar{\varepsilon}' Q_1 \bar{\varepsilon} \\ \frac{1}{n}\bar{\varepsilon}' Q_1 \varepsilon \end{bmatrix} = \begin{bmatrix} \sigma_\nu^2 \\ \sigma_\nu^2 \frac{1}{n} tr(W'W) \\ 0 \\ \sigma_1^2 \\ \sigma_1^2 \frac{1}{n} tr(W'W) \\ 0 \end{bmatrix} \tag{8}$$

where $\varepsilon = u - \rho\bar{u}$, $\bar{\varepsilon} = \bar{u} - \rho\bar{\bar{u}}$, $\bar{u} = (I_T \otimes W)u$, and $\bar{\bar{u}} = (I_T \otimes W)\bar{u}$. Dropping the expectation operator in (8) we have:

$$\begin{bmatrix} \frac{1}{n(T-1)}\varepsilon' Q_0 \varepsilon \\ \frac{1}{n(T-1)}\bar{\varepsilon}' Q_0 \bar{\varepsilon} \\ \frac{1}{n(T-1)}\bar{\varepsilon}' Q_0 \varepsilon \\ \frac{1}{n}\varepsilon' Q_1 \varepsilon \\ \frac{1}{n}\bar{\varepsilon}' Q_1 \bar{\varepsilon} \\ \frac{1}{n}\bar{\varepsilon}' Q_1 \varepsilon \end{bmatrix} = \begin{bmatrix} \sigma_\nu^2 \\ \sigma_\nu^2 \frac{1}{n} tr(W'W) \\ 0 \\ \sigma_1^2 \\ \sigma_1^2 \frac{1}{n} tr(W'W) \\ 0 \end{bmatrix} + \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \end{bmatrix} \tag{9}$$

The first set of GM estimators is based only on a subset of these moment conditions (the first three equations) and assigns equal weights to each of them:

$$(\tilde{\rho}, \tilde{\sigma}_\nu^2) = \arg\min_{\rho, \sigma_\nu^2} [\xi_1^2 + \xi_2^2 + \xi_3^2] \tag{10}$$

Using $\tilde{\rho}$ and $\tilde{\sigma}_\nu^2$ obtained from (10), an estimate for σ_1^2 can be obtained from the fourth equation as

$$\tilde{\sigma}_1^2 = \frac{1}{n}\tilde{\varepsilon}' Q_1 \tilde{\varepsilon} \tag{11}$$

This first set of estimators should be intended as initial and used to obtain the more efficient estimators described in the second set of GM estimators.

The second set of GM estimators uses all moment conditions and an optimal weighting scheme based on the inverse of the variance covariance matrix of the sample moments at the true parameter values:[7]

$$(\tilde{\rho}, \tilde{\sigma}_\nu^2, \tilde{\sigma}_1^2) = \arg\min_{\rho, \sigma_\nu^2, \sigma_1^2} [\xi' \widehat{VC}_\xi^{-1} \xi] \tag{12}$$

where

$$\underset{6\times 6}{VC_\xi} = \begin{bmatrix} \frac{1}{T-1}\sigma_\nu^4 & 0 \\ 0 & \sigma_1^4 \end{bmatrix} \otimes T_W, \tag{13}$$

and

$$T_W = \begin{bmatrix} 2 & 2tr(\frac{W'W}{n}) & 0 \\ 2tr(\frac{W'W}{n}) & 2tr(\frac{W'WW'W}{n}) & tr(\frac{W'W(W'+W)}{n}) \\ 0 & tr(\frac{W'W(W'+W)}{n}) & tr(\frac{WW+W'W}{n}) \end{bmatrix} \tag{14}$$

[7] Note that $\widehat{VC}_\xi$ is the same as VC_ξ except that σ_v^2 and σ_1^2 have been replaced by their estimates from the first set of GM estimators.

Kapoor *et al.* (2007) derive VC_ξ under the assumption of normally distributed innovations. They point out that, although the use of such a matrix is not strictly optimal in the absence of normality, it can be viewed as a reasonable approximation of the true and more complex variance covariance matrix.

The third set of GM estimators is motivated by computational difficulties. The elements of the asymptotic variance covariance matrix in (14) involve a computational count of up to $O(n^3)$. Although one could take advantage of the particular structure of W, the computation of such a matrix can still be challenging in many cases. The third set of GM estimators is identical to the second set except that it replaces T_W with an identity matrix I_3.

The model in Kapoor *et al.* (2007) does not include the spatial lag of the dependent variable. They prove that, under the assumptions made in their paper, OLS is a consistent estimator of β; and, thus, it can be used to calculate the estimated disturbances employed in the GM procedure. However, as it was shown in (7), because of the presence of the spatially lagged variable, OLS is no longer consistent and an instrumental variable approach is necessary.

Baltagi & Liu (2011) extend the instrumental variable estimator of Kelejian & Prucha (1998) to a random effects spatial autoregressive panel data model. They define four different estimators: a fixed effects spatial two stage least squares, a between effects spatial two stage least squares, a random effects spatial two stage least squares, and a spatial error component two stage least squares. The fixed effects spatial two stage least squares is also used by Mutl & Pfaffermayr (2011) to calculate the estimated disturbances employed in their GM procedure. Premultiplying equation (2) by Q_0 one obtains:

$$\tilde{y} = \tilde{Z}\delta + \tilde{u} \tag{15}$$

where $\tilde{Z} = Q_0 Z = Q_0[(I_T \otimes W)y, X] = [(I_T \otimes W)\tilde{y}, \tilde{X}]$. Applying the Kelejian & Prucha (1998) spatial two stage least squares procedure to this model, one gets the fixed effects spatial two stage least squares (FE-S2SLS) estimator of δ based on the instrument matrix $\tilde{H} = [\tilde{X}, (I_T \otimes W)\tilde{X}, (I_T \otimes W^2)\tilde{X}]$, that is:

$$\hat{\delta}_{FE-S2SLS} = (\hat{\tilde{Z}}'\tilde{Z})^{-1}(\hat{\tilde{Z}}'\tilde{y}) \tag{16}$$

An estimate of $Q_0 u$ can then be obtained from $\hat{\tilde{u}} = \tilde{y} - \tilde{Z}\hat{\delta}_{FE-S2SLS}$. Mutl

& Pfaffermayr (2011) formulate the first three moment conditions in (9) in terms of $Q_0 u$ and use the estimated residuals $\hat{\hat{u}}$ to obtain an estimate of ρ and σ_ν^2. With the solution of the first three moment conditions, they suggest to solve the fourth moment condition in (9) to estimate σ_ν^2. Note that this is only similar to the first set of GM estimators proposed by Kapoor *et al.* (2007). In fact, one should keep in mind that Kapoor *et al.* (2007) use OLS to estimate the regression equation, whereas Mutl & Pfaffermayr (2011) use an instrumental variables procedure on a within transformation of the model. As a result, their procedure does not allow them to consistently estimate $Q_1 u$ unless all the explanatory variables (other than the intercept) vary over time, since non-varying regressors are wiped out from the within transformation.[8]

To overcome this limitation, one can use the between effects spatial two stage least squares proposed in Baltagi & Liu (2011) to obtain an estimate of $Q_1 u$. In particular, premultipling equation (2) by Q_1 one obtains:

$$\bar{y} = \bar{Z}\delta + \bar{u} \tag{17}$$

where $\bar{Z} = Q_1 Z = Q_1[(I_T \otimes W)y, X] = [(I_T \otimes W)\bar{y}, \bar{X}]$. Applying the Kelejian & Prucha (1998) spatial two stage least squares procedure to this model, one gets the between effects spatial two stage least squares (BE-S2SLS) estimator of δ based on the instrument matrix $\bar{H} = [\bar{X}, (I_T \otimes W)\bar{X}, (I_T \otimes W^2)\bar{X}]$, that is:

$$\hat{\delta}_{BE-S2SLS} = (\hat{\bar{Z}}'\bar{Z})^{-1}(\hat{\bar{Z}}'\bar{y}) \tag{18}$$

An estimate of $Q_1 u$ can then be obtained from $\hat{\bar{u}} = \bar{y} - \bar{Z}\hat{\delta}_{BE-S2SLS}$. The last three moment conditions in Kapoor *et al.* (2007) can then be based on the estimated residuals $\hat{\bar{u}}$. Using these two vectors of residuals in the spatial GM procedure described in Kapoor *et al.* (2007), one obtains an estimate of the spatial parameter ρ and the two variance components σ_1^2 and σ_ν^2 using any of the three sets of GM estimators. It is important to stress that, contrary to Mutl & Pfaffermayr (2011), the suggested procedure enables one to use all six moment conditions. The Monte Carlo results show that the ability to use all six moment conditions leads to improved estimates of the spatial parameter and the variance components.

Finally, a feasible generalized spatial two stage least squares estimator of

[8] In a fixed effects model, one can recover the value of the intercept under the restriction that the sum of the individual effects is zero (see, Baltagi 2008, for details).

δ can be defined as:

$$\hat{\delta}_{FG2SLS} = (\hat{\check{Z}}'\check{Z})^{-1}\hat{\check{Z}}'\check{y} \tag{19}$$

where $\hat{\check{Z}} = P\check{Z}$, $P = H(H'H)^{-1}H'$, $H = [\tilde{H}, \bar{H}]$, $\check{Z} = (I_{nT} - \hat{\theta}Q_1)(I_T \otimes (I_n - \hat{\rho}W))Z$, $\check{y} = (I_{nT} - \hat{\theta}Q_1)(I_T \otimes (I_n - \hat{\rho}W))y$ and $\theta = 1 - \sigma_\nu/\sigma_1$. Statistical inference can be based on the following expression for the variance covariance matrix of the estimated parameters:

$$\text{Var}(\hat{\delta}_{FG2SLS}) = (\hat{\check{Z}}'\check{Z})^{-1} \tag{20}$$

The estimation procedure can be summarized as follows:

Step 1: Calculate the within effects spatial two stage least squares (FE-S2SLS) and obtain an estimate of Q_0u.

Step 2: Calculate the between effects spatial two stage least squares (BE-S2SLS) and obtain an estimate of Q_1u

Step 3: Use the estimated residuals from steps 1 and 2 to estimate the parameters ρ, σ_1^2 and σ_ν^2 by the GM procedure suggested in Kapoor *et al.* (2007)

Step 4: Use the estimates obtained in step 3 to perform a spatial Cochrane-Orcutt type transformation and the classical error component GLS transformation of the original model. Estimate the resulting model by two stage least squares using the matrix of instruments $H = [\tilde{H}, \bar{H}]$.

4 Monte Carlo experiments

In this section, first a Monte Carlo model is specified, and then results are given which suggest that the procedure is effective in small samples. The experimental design for the Monte Carlo simulation is based on the format extensively used in studies on spatial panel regression models. The Monte Carlo study considers the three suggested GM estimators of the spatial autoregressive parameter and the variance components, and the corresponding feasible GLS estimator for the parameters of the regression equation. For purposes of comparison, the estimation procedure suggested by Mutl & Pfaffermayr (2011) (henceforth MP) is also considered, as well as a maximum likelihood estimator (Millo & Piras 2012) (henceforth ML).

4.1 Monte Carlo model

The design of the Monte Carlo experiment draws on previous studies in spatial econometrics (Kelejian & Prucha 1999, Kelejian *et al.* 2004, Kapoor *et al.* 2007, Baltagi *et al.* 2003, 2007a,c, Kelejian & Piras 2011). In all Monte Carlo experiments, the data are generated according to the following model:

$$y = \lambda(I_T \otimes W)y + X\beta + u \tag{21}$$

where u follows the autoregressive model described in equations (3) and (4), that is:

$$u = \rho(I_T \otimes W)u + \varepsilon \tag{22}$$

and

$$\varepsilon = (\iota_T \otimes I_N)\mu + \nu \tag{23}$$

and, for simplicity, it is assumed that the two spatial weighting matrices are the same.[9] In equation (21), the regressor matrix is taken as $X = (e_n, x_1, x_2)$, where e_n is an $nT \times 1$ vector of unit elements, x_1 is allowed to vary both over time and cross-section, and x_2 is time-invariant and is allowed to vary only over the cross-sectional units. In particular, the values of x_2 are generated as a random sample from the uniform distribution over (0,5). As for x_1, the first n values (say x_{11}) are generated as a random sample from the uniform distribution over (0,5). The remaining four cross section are obtained as

$$x_{1i} = x_{11} + \xi \tag{24}$$

where ξ is a normally distributed random variable of zero mean and variance one. Note that, as a result of this strategy in generating x_1, the between variation is about three times larger than the within variation, which is usually the case for large n and small T (i.e. micro panel).[10]

The vector of parameters β is taken to be equal to 1. Two sets of data are generated, corresponding to two regular grids of dimension 10×10 and 15×15, leading to sample sizes of, respectively, $n = 100$ and $n = 225$ observations. Only one time dimension, namely $T = 5$, is considered. For each sample size, three row normalized weighting matrices are defined. These

[9] Although restrictive, this assumption is generally made in many spatial econometrics applications (e.g. Donovan *et al.* 2007, Arraiz *et al.* 2010, Piras & Lozano-Gracia 2012).

[10] The experiment was designed with a target R^2 value in mind of, roughly, 0.6 (for a combination of ρ=λ=0.2).

matrices differ in their degree of sparseness. Following Kelejian & Prucha (1999), these matrices are defined in a circular world and they are generally referred to as "k ahead and k behind" spatial weighting matrices. In the first weighting matrix (W_1), k is set to 2; and, therefore, the non-zero elements in row 1 and N are, respectively, (1,2), (1,3), (1,N-1), (1,N) and (N,1), (N,2), (N,N-2), (N,N-1). The second (W_2) and third (W_3) matrices are defined in a similar fashion, and k is set to 6 and 10 respectively. In all of the Monte Carlo experiments $\sigma_\mu^2 = 1$ and $\sigma_\nu^2 = 1$. Given the selection for T, $\sigma_1^2 = 6$. Nine values are considered for ρ, namely, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6 and 0.8; and five for λ, namely, -0.6, -0.2, 0, 0.2, 0.6. As for the specification of the disturbance term, the vectors μ and ν are specified to be independent, and normally distributed. The elements of μ and ν are, respectively, *i.i.d.* $N(0, \sigma_\mu^2)$ and *i.i.d.* $N(0, \sigma_v^2)$.

The setup then amounts to a total of 270 experiments resulting from nine different values of ρ, five different values for λ, two different sample sizes, and the choice of three spatial weighting matrices. For each of these experiments, the estimator proposed in Mutl & Pfaffermayr (2011), the maximum likelihood estimator and the three sets of GM estimators proposed in this paper are computed. For each experiment, 1,000 replications are performed.

4.2 Monte Carlo results

This section presents the results of the Monte Carlo experiments. Following Kapoor *et al.* (2007), the adopted measure of dispersion is related to the standard measure of the root mean squared error, but based on quantiles rather than moments. This measure is defined as:

$$\text{RMSE} = \left[bias^2 + \left[\frac{IQ}{1.35} \right]^2 \right]^{1/2} \tag{25}$$

where *bias* is the difference between the median and the true parameter value, and IQ is the interquantile range defined as $q_3 - q_1$ where q_3 is the 0.75 quantile and q_1 is the 0.25 quantile.

Tables 1 - 7 summarize the main evidence for $N = 100$ observations.[11] The figures relate to the RMSE calculated using (25). The tables contain

[11] Results for the larger sample size ($N = 225$) are qualitatively similar and, therefore, are not reported. They are available upon request from the author.

the results based on two of the three weighting matrices: "2 ahead and 2 behind" (columns 3 - 7), and "6 ahead and 6 behind" (columns 8 - 12).[12] For each weighting matrix, the five columns in the tables correspond to different estimators. The first three columns report results based on the three sets of GM estimators. In particular, the first column (IN) is the initial estimator, the second column (FW) corresponds to the second set of GM estimators, and the third column (W) corresponds to the third set of GM estimators (i.e. the simplified weighting scheme). As for the last two columns, they display the results obtained using the estimation procedure in Mutl & Pfaffermayr (2011) (ML) and a maximum likelihood estimators (ML). Following the same structure, table 2 reports the results for σ_ν^2, table 3 contains the results for σ_1^2, table 4 is devoted to θ, table 5 display results on λ and, finally, tables 6 and 7 report the results for, β_1 and β_2, respectively.

Table 1 reveals that the RMSEs of ρ calculated using MP is, on average, 19% larger than that of the weighted GM estimators, and 35% larger than that of the ML estimator when the spatial weighting matrix is "2 ahead and 2 behind". Interestingly, this difference drops to slightly more than 16% and 32%, respectively, when the spatial weighting matrix employed is "6 ahead and 6 behind". On the other hand, the difference between the ML and GM is about 14% when the spatial weighting matrix is "2 ahead and 2 behind", and 13% when the spatial weighting matrix is "6 ahead and 6 behind".

Furthermore, it should be noted that the MP estimator of ρ corresponds to the initial GM estimator (IN) of Kapoor *et al.* (2007); and, therefore, the previous result is not surprising. Simulations results in Kapoor *et al.* (2007) showed that, on average, the RMSEs of the unweighted estimator of ρ was 17% larger than that of the weighted GM estimator. On the other hand, results relating to the weighted and partially weighted GM estimators are only slightly different (with differences that range between 3% and 4%). The computational benefits related to the partially weighted estimator are associated only with a small cost in terms of efficiency. This also reveals that the variance factors are important in determining the efficiency of the GM estimators, probably even more than the covariances structure (given the choices for the spatial weighting matrices). The variance factors in the variance covariance weighting matrix in (13) are $\sigma_\nu^4/(T-1)$ and σ_1^4. Given

[12] Results for the spatial weighting matrix defined as "15 ahead and 15 behind" are qualitatively very similar and, therefore, are left out of the paper. They are available from the author.

our setup, these variance factors are, respectively, 0.25 and 36, implying a ratio of 0.25/36= 0.069. It is also interesting to note that the figures for the RMSEs increase proportionally with the degree of sparseness of the weighting matrix. When the weighting matrix is “2 ahead and 2 behind”, the average RMSE for all four methods is 0.326. The same average increases to 0.599 when the weighting matrix is “6 ahead and 6 behind”.

In table 2, we present the results on the RMSEs for σ_ν^2. Again, the first and fourth columns are the same because the MP estimator of σ_ν^2 corresponds to the initial GM estimator (IN) of Kapoor *et al.* (2007). The columns averages on the last line of the table show that there is little difference between the initial estimators and any of the weighted versions of the GM estimators. As an example, when the spatial weighting matrix is “2 ahead and 2 behind”, the difference between FW and MP is, on average, only about 3%. The ML produces, on average, the lowest RMSE. The difference with the GM estimator is 7%, 8.5% and almost 11%. Consistent with the evidence for ρ, differences between the various estimators become irrelevant when the spatial structure becomes denser. At the same time, though, the RMSE seems not to depend much on the degree of sparseness of the weighting matrices either.

The results on σ_1^2 presented in table 3 prove that, when some of the regressors in the model do not vary over time, RMSEs obtained using the MP procedure can be very large. On the other hand, using the Baltagi & Liu (2011) between effects spatial two stage least square estimator leads to a consistent estimate of the vector of residuals $Q_1 u$, which, in turn, leads to a smaller RMSE. Again there is no substantial loss in terms of efficiency if the model is estimated with either the initial or the partially weighted GM estimators (RMSEs obtained by using the initial estimator are, on average, only 1% higher than those from the fully weighted GM estimator). The same holds for the RMSEs obtained using the ML estimator since the difference is in the order of 2%.

The differences in terms of the RMSEs for σ_1^2 also influence the estimates of $\theta = 1 - \sigma_\nu/\sigma_1$ (presented in table 4). The RMSEs associated with the MP estimation of θ are almost 45% higher than those obtained estimating the model by any of the three GM estimators or the ML. On the other hand, there are very small differences between the three GM estimators and the ML estimator for all weighting matrices considered.

Finally, tables 5 to 7 report the RMSEs for the feasible GLS estimators of λ, β_1, and β_2. These tables reveal that the RMSEs of the various feasible

GLS estimators are all very similar. At the same time, the RMSEs of MP are higher than those obtained with the other methods. The RMSEs obtained by the ML estimator are, on average, consistently lower than any of the GM estimators. However, the results are encouraging because these differences are quite small.

5 Conclusions

The present paper introduces an estimation procedure for a Cliff and Ord type panel data model with random effects. The proposed procedure provides an improvement over the existing one in, at least, two ways. On the one hand, our estimation procedure considers all of the moment conditions in Kapoor *et al.* (2007). On the other hand, it handles the case of time-invariant regressors without losing efficiency. The Monte Carlo results presented in the paper have demonstrated that the procedure is very effective also in small samples. Furthermore, our Monte Carlo results also show that the procedure compares well to the ML.

References

Anselin, L., Le Gallo, J., & Jayet, H. 2008. Spatial Panel Econometrics. *Pages 624 – 660 of:* Matyas, L., & Sevestre, P. (eds), *The econometrics of Panel Data, Fundamentals and Recent Developments in Theory and Practice (3rd Edition)*. Springer-Verlag, Berlin Heidelberg.

Arraiz, I., Drukker, D.M., Kelejian, H.H., & Prucha, I.R. 2010. A spatial Cliff-Ord-type Model with Heteroscedastic Innovations: Small and Large Sample Results. *Journal of Regional Science*, **50**, 592 – 614.

Baltagi, B.H. 2008. *Econometric Analysis of Panel Data, 4th edition.* New York: Wiley.

Baltagi, B.H., & Liu, L. 2008. Testing for random effects and spatial lag dependence in panel data models. *Statistics and Probability Letters*, **78**, 3304–3306.

Baltagi, B.H., & Liu, L. 2011. Instrumental Variable Estimation of a Spatial

Autoregressive Panel Model with Random Effects. *Economics Letters*, **111**, 135–137.

Baltagi, B.H., Song, S.H., & Koh, W. 2003. Testing panel data regression models with spatial error correlation. *Journal of Econometrics*, **117**, 123–150.

Baltagi, B.H., Kelejian, H.H., & Prucha, I.R. 2007a. Analysis of spatially dependent data. *Journal of Econometrics*, **140**, 1–4.

Baltagi, B.H., Egger, P., & Pfaffermayr, M. 2007b. Estimating models of complex FDI: are there third-country effects? *Journal of Econometrics*, **140**, 260–281.

Baltagi, B.H., Song, S.H., Jung, B.C., & Koh, W. 2007c. Testing for serial correlation, spatial autocorrelation and random effects using panel data. *Journal of Econometrics*, **140**(1), 5–51.

Debarsy, N., & Ertur, C. 2010. Testing for spatial autocorrelation in a fixed effects panel data model. *Regional Science and Urban Economics*, **40**, 453–470.

Donovan, G. H., Champ, P. A., & Butry, D. T. 2007. Wildfire Risk and Housing Prices: A Case Study from Colorado Springs. *Land Economics*, **83**(3), 217–233.

Elhorst, J.P. 2003. Specification and estimation of spatial panel data models. *International Regional Sciences Review*, **26**(3), 244–268.

Elhorst, J.P. 2008. Serial and Spatial error correlation. *Economics Letters*, **100**, 422–424.

Elhorst, J.P. 2009. Spatial Panel Data Models. *In:* Fischer, M. M., & Getis, A. (eds), *Handbook of Applied Spatial Analysis*. Springer, Berlin, Heidelberg, New York.

Elhorst, J.P. 2010. Dynamic panels with endogenous interactions effects when T is small. *Regional Science and Urban Economics*, **40**, 272–282.

Elhorst, J.P., & Freret, S. 2009. Yardstick competition among local governments: French evidence using a two -regimes spatial panel data model. *Journal of Regional Science*, **49**, 931–951.

Elhorst, J.P., Piras, G., & Arbia, G. 2010. Growth and Convergence in a multi-regional model with space-time dynamics. *Geographical Analysis*, **42**, 338–355.

Fingleton, B. 2008. A Generalized Method of Moments Estimator for a Spatial Panel Model with an Endogenous Spatial Lag and Spatial Moving Average Errors. *Spatial Economic Analysis*, **3**(1), 27–44.

Fingleton, B., & Le Gallo, J. 2008. Estimating spatial models with endogenous variables, a spatial lag and spatially dependent disturbances: Finite sample properties. *Papers in Regional Science*, **87**(3), 319–340.

Kapoor, M., Kelejian, H.H., & Prucha, I.R. 2007. Panel data model with spatially correlated error components. *Journal of Econometrics*, **140**(1), 97–130.

Kelejian, H.H., & Piras, G. 2011. An extension of Kelejian's J-test for non-nested spatial models. *Regional Science and Urban Economics*, **41**, 281–292.

Kelejian, H.H., & Prucha, I.R. 1998. A Generalized Spatial Two Stages Least Square Procedure for Estimating a Spatial Autoregressive Model with Autoregressive Disturbances. *Journal of Real Estate Finance and Economics*, **17**(1), 99–121.

Kelejian, H.H., & Prucha, I.R. 1999. A Generalized Moments Estimator for the Autoregressive Parameter in a Spatial Model. *International Economic Review*, **40**(2), 509–533.

Kelejian, H.H., Prucha, I.R., & Yuzefovich, Y. 2004. Instrumental variable estimation of a spatial autoregressive model with autoregressive disturbances: Large and small sample results. *Pages 163–198 of:* LeSage, J. P., & Pace, R.K. (eds), *Advances in Econometrics: Spatial and Spatio-Temporal econometrics*. Elsevier Sciences Ltd., Oxford, U.K.

Lee, L.F., & Yu, J. 2010a. A spatial dynamic panel data model with both time and individual fixed effects. *Econometric Theory*, **26**, 564–597.

Lee, L.F., & Yu, J. 2010b. Estimation of spatial autoregressive panel data models with fixed effects. *Journal of Econometrics*, **154**, 165–185.

Lee, L.F., & Yu, J. 2010c. Some recent development in spatial panel data models. *Regional Science and Urban Economics*, **40**, 255–271.

Millo, G., & Piras, G. 2012. splm: Spatial Panel Data Models in R. *Journal of Statistical Software*, **47**(1), 1–38.

Mutl, J. 2006. *Dynamic panel data models with spatially autocorrelated disturbances*. PhD Thesis, University of Maryland, College Park.

Mutl, J., & Pfaffermayr, M. 2011. The Hausman test in a Cliff and Ord panel model. *Econometrics Journal*, **14**, 48–76.

Parent, O., & LeSage, J.P. 2010. A spatial dynamic panel model with random effects applied to commuting times. *Transportation Research Part B*, **44**, 633–645.

Pesaran, H.M., & Tosetti, E. 2011. Large panels with common factors and spatial correlations. *Journal of Econometrics*, **161**(2), 182 –202.

Piras, G., & Lozano-Gracia, N. 2012. Spatial J-test: some Monte Carlo evidence. *Statistics and Computing*, **22**, 169–183.

Yu, J., & Lee, L.F. 2010. Estimation of unit root spatial dynamic panel data models. *Econometric Theory*, **26**, 1332–1362.

Yu, J., de Jong, R., & Lee, L.F. 2008. Quasi maximum likelihood estimators for spatial dynamic panel data with fixed effects when both n and T are large. *Journal of Econometrics*, **146**, 118–134.

Table (1) RMSEs of ρ using Mutl & Pfaffermayr (2011), the three sets of GM estimators, and the ML estimator.

		2 ahead and 2 behind					6 ahead and 6 behind				
		IN	FW	W	MP	ML	IN	FW	W	MP	ML
$\rho=-0.8$	$\lambda=-0.6$	0.2928	0.2857	0.2860	0.2928	0.2831	0.3425	0.3427	0.3338	0.3425	0.3698
$\rho=-0.6$	$\lambda=-0.6$	0.3603	0.3407	0.3256	0.3603	0.2922	0.6140	0.6136	0.5997	0.6140	0.5848
$\rho=-0.4$	$\lambda=-0.6$	0.4761	0.3995	0.3932	0.4761	0.3210	0.7506	0.6560	0.6574	0.7506	0.5379
$\rho=-0.2$	$\lambda=-0.6$	0.3038	0.3222	0.3193	0.3038	0.3288	0.6640	0.5701	0.6006	0.6640	0.5523
$\rho=0$	$\lambda=-0.6$	0.3530	0.2514	0.2548	0.3530	0.1526	0.7107	0.5954	0.5750	0.7107	0.3092
$\rho=0.2$	$\lambda=-0.6$	0.2922	0.2831	0.2818	0.2922	0.2748	0.3620	0.3586	0.3527	0.3620	0.3884
$\rho=0.4$	$\lambda=-0.6$	0.3551	0.3062	0.3019	0.3551	0.2895	0.6221	0.5959	0.6117	0.6221	0.5870
$\rho=0.6$	$\lambda=-0.6$	0.3579	0.3581	0.3541	0.3579	0.3163	0.7543	0.6810	0.6979	0.7543	0.5167
$\rho=0.8$	$\lambda=-0.6$	0.3178	0.3228	0.3035	0.3178	0.2523	0.8143	0.5976	0.5771	0.8143	0.5161
$\rho=-0.8$	$\lambda=-0.2$	0.3570	0.2488	0.2482	0.3570	0.1774	0.6470	0.5224	0.5225	0.6470	0.4302
$\rho=-0.6$	$\lambda=-0.2$	0.3026	0.2883	0.2916	0.3026	0.2732	0.3544	0.3507	0.3357	0.3544	0.3762
$\rho=-0.4$	$\lambda=-0.2$	0.3517	0.2988	0.3290	0.3517	0.3296	0.6383	0.6337	0.6229	0.6383	0.6659
$\rho=-0.2$	$\lambda=-0.2$	0.3734	0.3263	0.3097	0.3734	0.3231	0.6578	0.5632	0.5514	0.6578	0.5248
$\rho=0$	$\lambda=-0.2$	0.3922	0.3040	0.2962	0.3922	0.2830	0.6392	0.5490	0.5347	0.6392	0.4986
$\rho=0.2$	$\lambda=-0.2$	0.3352	0.2779	0.2738	0.3352	0.2029	0.6360	0.5911	0.5989	0.6360	0.4489
$\rho=0.4$	$\lambda=-0.2$	0.2819	0.2764	0.2784	0.2819	0.2499	0.3506	0.3476	0.3418	0.3506	0.3737
$\rho=0.6$	$\lambda=-0.2$	0.3932	0.3091	0.3175	0.3932	0.3255	0.6200	0.6189	0.6113	0.6200	0.5761
$\rho=0.8$	$\lambda=-0.2$	0.4071	0.2938	0.3539	0.4071	0.2549	0.7563	0.6790	0.7341	0.7563	0.6441
$\rho=-0.8$	$\lambda=0$	0.3235	0.2958	0.3016	0.3235	0.2700	0.7812	0.6284	0.6145	0.7812	0.5120
$\rho=-0.6$	$\lambda=0$	0.3428	0.3186	0.3211	0.3428	0.2655	0.8517	0.6407	0.6489	0.8517	0.5361
$\rho=-0.4$	$\lambda=0$	0.2891	0.2779	0.2807	0.2891	0.2597	0.3395	0.3391	0.3330	0.3395	0.3804
$\rho=-0.2$	$\lambda=0$	0.4191	0.3105	0.3567	0.4191	0.2634	0.6240	0.6158	0.6111	0.6240	0.5156
$\rho=0$	$\lambda=0$	0.3354	0.2733	0.2960	0.3354	0.2590	0.7652	0.6456	0.6874	0.7652	0.5364
$\rho=0.2$	$\lambda=0$	0.3497	0.2787	0.2938	0.3497	0.2722	0.8181	0.6315	0.8230	0.8181	0.5540
$\rho=0.4$	$\lambda=0$	0.3690	0.3020	0.3023	0.3690	0.2847	0.7493	0.6865	0.6744	0.7493	0.3884
$\rho=0.6$	$\lambda=0$	0.2947	0.2790	0.2821	0.2947	0.2266	0.3518	0.3432	0.3427	0.3518	0.3772
$\rho=0.8$	$\lambda=0$	0.4128	0.3234	0.3085	0.4128	0.3212	0.6321	0.6305	0.6252	0.6321	0.6797
$\rho=-0.8$	$\lambda=0.2$	0.4103	0.3205	0.3348	0.4103	0.2815	0.7578	0.5708	0.7523	0.7578	0.5178
$\rho=-0.6$	$\lambda=0.2$	0.3391	0.3160	0.3170	0.3391	0.2795	0.7990	0.7082	0.7857	0.7990	0.6207
$\rho=-0.4$	$\lambda=0.2$	0.3802	0.3282	0.3276	0.3802	0.2803	0.9158	0.7063	0.7148	0.9158	0.5053
$\rho=-0.2$	$\lambda=0.2$	0.2854	0.2739	0.2758	0.2854	0.2540	0.3500	0.3481	0.3417	0.3500	0.3787
$\rho=0$	$\lambda=0.2$	0.3925	0.3036	0.3330	0.3925	0.2931	0.6374	0.6245	0.6234	0.6374	0.5218
$\rho=0.2$	$\lambda=0.2$	0.3915	0.3167	0.3456	0.3915	0.3230	0.7827	0.6137	0.6446	0.7827	0.6662
$\rho=0.4$	$\lambda=0.2$	0.3813	0.3566	0.3688	0.3813	0.2777	0.8909	0.6672	0.7142	0.8909	0.5920
$\rho=0.6$	$\lambda=0.2$	0.4065	0.3314	0.3321	0.4065	0.2727	0.8945	0.6673	0.7220	0.8945	0.5678
$\rho=0.8$	$\lambda=0.2$	0.2849	0.2579	0.2770	0.2849	0.2239	0.3465	0.3366	0.3378	0.3465	0.3723
$\rho=-0.8$	$\lambda=0.6$	0.4657	0.2982	0.3261	0.4657	0.2481	0.6348	0.6287	0.6240	0.6348	0.4897
$\rho=-0.6$	$\lambda=0.6$	0.4077	0.3150	0.3248	0.4077	0.2726	0.7734	0.5948	0.6987	0.7734	0.4990
$\rho=-0.4$	$\lambda=0.6$	0.4012	0.3100	0.3477	0.4012	0.2742	0.9237	0.6182	0.6603	0.9237	0.5289
$\rho=-0.2$	$\lambda=0.6$	0.5742	0.4511	0.4618	0.5742	0.2996	0.8251	0.6882	0.7165	0.8251	0.5701
$\rho=0$	$\lambda=0.6$	0.2826	0.2279	0.2750	0.2826	0.2019	0.3470	0.3317	0.3277	0.3470	0.3631
$\rho=0.2$	$\lambda=0.6$	0.3670	0.2895	0.3304	0.3670	0.2461	0.6448	0.5790	0.6377	0.6448	0.5208
$\rho=0.4$	$\lambda=0.6$	0.3828	0.3293	0.3545	0.3828	0.2387	0.7769	0.6537	0.7769	0.7769	0.5167
$\rho=0.6$	$\lambda=0.6$	0.4457	0.3340	0.3906	0.4457	0.2726	0.8611	0.7533	0.8092	0.8611	0.5462
$\rho=0.8$	$\lambda=0.6$	0.4903	0.3289	0.3348	0.4903	0.2717	0.9502	0.5208	0.5860	0.9502	0.4775
	Average	0.3673	0.3076	0.3182	0.3673	0.2703	0.6657	0.5698	0.5932	0.6657	0.5030

Note: N = 100, T = 5, and K = 2.

Table (2) RMSEs of σ_ν^2 using Mutl & Pfaffermayr (2011), the three sets of GM estimators, and the ML estimator.

		2 ahead and 2 behind					6 ahead and 6 behind				
		IN	FW	W	MP	ML	IN	FW	W	MP	ML
$\rho=-0.8$	$\lambda=-0.6$	0.1899	0.1889	0.1981	0.1899	0.1890	0.1826	0.1837	0.1862	0.1826	0.1809
$\rho=-0.6$	$\lambda=-0.6$	0.2008	0.2033	0.2024	0.2008	0.1841	0.2132	0.2158	0.2145	0.2132	0.2196
$\rho=-0.4$	$\lambda=-0.6$	0.2512	0.2456	0.2490	0.2512	0.2223	0.1913	0.1885	0.1893	0.1913	0.1686
$\rho=-0.2$	$\lambda=-0.6$	0.2331	0.2241	0.2225	0.2331	0.2132	0.2138	0.2114	0.2120	0.2138	0.1893
$\rho=0$	$\lambda=-0.6$	0.2827	0.2586	0.2744	0.2827	0.1959	0.2414	0.2350	0.2377	0.2414	0.1952
$\rho=0.2$	$\lambda=-0.6$	0.2073	0.2117	0.2119	0.2073	0.2183	0.1876	0.1862	0.1851	0.1876	0.1879
$\rho=0.4$	$\lambda=-0.6$	0.1998	0.1997	0.1994	0.1998	0.2061	0.1815	0.1780	0.1810	0.1815	0.1735
$\rho=0.6$	$\lambda=-0.6$	0.1992	0.2002	0.1991	0.1992	0.1998	0.2115	0.2049	0.2047	0.2115	0.2010
$\rho=0.8$	$\lambda=-0.6$	0.2471	0.2274	0.2366	0.2471	0.1908	0.2130	0.2130	0.2125	0.2130	0.1848
$\rho=-0.8$	$\lambda=-0.2$	0.3016	0.2786	0.2849	0.3016	0.2551	0.2141	0.2096	0.2120	0.2141	0.1937
$\rho=-0.6$	$\lambda=-0.2$	0.2072	0.2197	0.2100	0.2072	0.2298	0.1803	0.1829	0.1820	0.1803	0.1827
$\rho=-0.4$	$\lambda=-0.2$	0.1809	0.1809	0.1833	0.1809	0.1765	0.1991	0.2008	0.1993	0.1991	0.2004
$\rho=-0.2$	$\lambda=-0.2$	0.2147	0.2104	0.2112	0.2147	0.1836	0.2355	0.2346	0.2340	0.2355	0.1994
$\rho=0$	$\lambda=-0.2$	0.2589	0.2509	0.2532	0.2589	0.2248	0.1779	0.1775	0.1764	0.1779	0.1768
$\rho=0.2$	$\lambda=-0.2$	0.2764	0.2457	0.2681	0.2764	0.2391	0.2606	0.2553	0.2585	0.2606	0.2254
$\rho=0.4$	$\lambda=-0.2$	0.1979	0.1789	0.1874	0.1979	0.1778	0.2023	0.2015	0.2029	0.2023	0.2153
$\rho=0.6$	$\lambda=-0.2$	0.1767	0.1789	0.1785	0.1767	0.1761	0.1746	0.1747	0.1733	0.1746	0.1723
$\rho=0.8$	$\lambda=-0.2$	0.1778	0.1781	0.1773	0.1778	0.1812	0.1909	0.1949	0.1924	0.1909	0.2021
$\rho=-0.8$	$\lambda=0$	0.1783	0.1792	0.1789	0.1783	0.1816	0.1984	0.1993	0.1996	0.1984	0.2043
$\rho=-0.6$	$\lambda=0$	0.2864	0.2630	0.2773	0.2864	0.2210	0.2425	0.2359	0.2391	0.2425	0.2089
$\rho=-0.4$	$\lambda=0$	0.1981	0.1898	0.1970	0.1981	0.1903	0.2446	0.2472	0.2446	0.2446	0.2486
$\rho=-0.2$	$\lambda=0$	0.1790	0.1795	0.1768	0.1790	0.1797	0.1722	0.1764	0.1735	0.1722	0.1722
$\rho=0$	$\lambda=0$	0.2177	0.2192	0.2190	0.2177	0.2239	0.2084	0.2047	0.2071	0.2084	0.1996
$\rho=0.2$	$\lambda=0$	0.1958	0.1899	0.1910	0.1958	0.1873	0.2010	0.2016	0.2021	0.2010	0.1826
$\rho=0.4$	$\lambda=0$	0.3174	0.2956	0.3096	0.3174	0.2008	0.2062	0.1998	0.2034	0.2062	0.1896
$\rho=0.6$	$\lambda=0$	0.2751	0.2545	0.2766	0.2751	0.2309	0.2007	0.2057	0.2036	0.2007	0.1999
$\rho=0.8$	$\lambda=0$	0.1954	0.1926	0.1934	0.1954	0.1826	0.2253	0.2185	0.2250	0.2253	0.2133
$\rho=-0.8$	$\lambda=0.2$	0.2182	0.2185	0.2198	0.2182	0.2098	0.2219	0.2242	0.2236	0.2219	0.2170
$\rho=-0.6$	$\lambda=0.2$	0.1759	0.1683	0.1687	0.1759	0.1604	0.2012	0.1995	0.1987	0.2012	0.1999
$\rho=-0.4$	$\lambda=0.2$	0.2602	0.2476	0.2510	0.2602	0.2227	0.2016	0.1981	0.2011	0.2016	0.1724
$\rho=-0.2$	$\lambda=0.2$	0.2284	0.2072	0.2147	0.2284	0.1852	0.1738	0.1739	0.1768	0.1738	0.1756
$\rho=0$	$\lambda=0.2$	0.2198	0.2210	0.2190	0.2198	0.2075	0.1814	0.1795	0.1824	0.1814	0.1819
$\rho=0.2$	$\lambda=0.2$	0.2022	0.1993	0.2017	0.2022	0.1917	0.1948	0.1955	0.1945	0.1948	0.1982
$\rho=0.4$	$\lambda=0.2$	0.2055	0.2064	0.2066	0.2055	0.1991	0.2111	0.2106	0.2099	0.2111	0.2114
$\rho=0.6$	$\lambda=0.2$	0.2358	0.2214	0.2300	0.2358	0.2221	0.1969	0.2034	0.2037	0.1969	0.2089
$\rho=0.8$	$\lambda=0.2$	0.2187	0.2159	0.2186	0.2187	0.1885	0.2218	0.2225	0.2211	0.2218	0.2214
$\rho=-0.8$	$\lambda=0.6$	0.2341	0.2287	0.2370	0.2341	0.2123	0.2464	0.2498	0.2487	0.2464	0.2575
$\rho=-0.6$	$\lambda=0.6$	0.2101	0.2116	0.2105	0.2101	0.2084	0.1864	0.1879	0.1871	0.1864	0.1844
$\rho=-0.4$	$\lambda=0.6$	0.1996	0.1998	0.2017	0.1996	0.1814	0.2177	0.2193	0.2191	0.2177	0.2195
$\rho=-0.2$	$\lambda=0.6$	0.2249	0.2173	0.2139	0.2249	0.2235	0.1987	0.1964	0.1963	0.1987	0.2053
$\rho=0$	$\lambda=0.6$	0.2480	0.2257	0.2293	0.2480	0.2117	0.2000	0.1862	0.1881	0.2000	0.1809
$\rho=0.2$	$\lambda=0.6$	0.2212	0.2165	0.2158	0.2212	0.1979	0.2121	0.2024	0.2029	0.2121	0.1930
$\rho=0.4$	$\lambda=0.6$	0.2061	0.1971	0.2049	0.2061	0.1862	0.2343	0.2249	0.2274	0.2343	0.2083
$\rho=0.6$	$\lambda=0.6$	0.2400	0.2351	0.2405	0.2400	0.2216	0.1804	0.1798	0.1791	0.1804	0.1896
$\rho=0.8$	$\lambda=0.6$	0.1852	0.1919	0.1955	0.1852	0.1881	0.2205	0.2163	0.2187	0.2205	0.1916
	Average	0.2218	0.2150	0.2188	0.2218	0.2018	0.2060	0.2046	0.2051	0.2060	0.1979

Note: N = 100, T = 5, and K = 2.

Table (3) RMSEs of σ_1^2 using Mutl & Pfaffermayr (2011), the three sets of GM estimators, and the ML estimator.

		2 ahead and 2 behind					6 ahead and 6 behind				
		IN	FW	W	MP	ML	IN	FW	W	MP	ML
$\rho = -0.8$	$\lambda = -0.6$	0.3964	0.3850	0.3916	2.4072	0.3985	0.4092	0.4073	0.4081	2.3656	0.3877
$\rho = -0.6$	$\lambda = -0.6$	0.4284	0.4252	0.4207	2.3646	0.4321	0.4249	0.4215	0.4196	2.3214	0.4204
$\rho = -0.4$	$\lambda = -0.6$	0.4426	0.4368	0.4437	2.2780	0.4288	0.4648	0.4688	0.4667	2.3306	0.4946
$\rho = -0.2$	$\lambda = -0.6$	0.4466	0.4376	0.4409	2.3289	0.4459	0.4453	0.4475	0.4443	2.3487	0.4404
$\rho = 0$	$\lambda = -0.6$	0.4953	0.4874	0.4914	2.5178	0.4661	0.4542	0.4441	0.4463	2.4657	0.4004
$\rho = 0.2$	$\lambda = -0.6$	0.4469	0.4330	0.4373	2.3857	0.4155	0.4093	0.4157	0.4107	2.3265	0.3876
$\rho = 0.4$	$\lambda = -0.6$	0.4796	0.4710	0.4759	2.2737	0.4698	0.4462	0.4418	0.4398	2.2869	0.4427
$\rho = 0.6$	$\lambda = -0.6$	0.4154	0.4109	0.4147	2.3169	0.3857	0.4439	0.4425	0.4423	2.3047	0.4217
$\rho = 0.8$	$\lambda = -0.6$	0.4497	0.4376	0.4436	2.2780	0.3856	0.4437	0.4375	0.4375	2.3264	0.4349
$\rho = -0.8$	$\lambda = -0.2$	0.4628	0.4524	0.4587	2.5588	0.4143	0.4708	0.4593	0.4588	2.4142	0.4557
$\rho = -0.6$	$\lambda = -0.2$	0.4217	0.4187	0.4209	2.4021	0.3978	0.4674	0.4646	0.4643	2.2899	0.4676
$\rho = -0.4$	$\lambda = -0.2$	0.4510	0.4514	0.4557	2.2699	0.4700	0.4291	0.4368	0.4314	2.2792	0.4359
$\rho = -0.2$	$\lambda = -0.2$	0.4707	0.4670	0.4631	2.3219	0.4705	0.4298	0.4268	0.4265	2.3204	0.4253
$\rho = 0$	$\lambda = -0.2$	0.4309	0.4207	0.4272	2.3045	0.4042	0.4440	0.4408	0.4401	2.3181	0.4424
$\rho = 0.2$	$\lambda = -0.2$	0.4610	0.4775	0.4695	2.5027	0.4401	0.4754	0.4607	0.4641	2.5057	0.4538
$\rho = 0.4$	$\lambda = -0.2$	0.4281	0.4157	0.4139	2.4962	0.4066	0.4390	0.4329	0.4329	2.3054	0.4462
$\rho = 0.6$	$\lambda = -0.2$	0.4365	0.4275	0.4347	2.2731	0.4316	0.4734	0.4683	0.4673	2.3296	0.4571
$\rho = 0.8$	$\lambda = -0.2$	0.4611	0.4548	0.4562	2.2832	0.4594	0.4307	0.4280	0.4287	2.2612	0.4132
$\rho = -0.8$	$\lambda = 0$	0.4160	0.4062	0.4085	2.3331	0.4091	0.4587	0.4510	0.4536	2.2851	0.4435
$\rho = -0.6$	$\lambda = 0$	0.5220	0.5230	0.5148	2.4386	0.5100	0.4361	0.4277	0.4281	2.4262	0.4018
$\rho = -0.4$	$\lambda = 0$	0.4254	0.4252	0.4350	2.4094	0.4403	0.4577	0.4616	0.4569	2.3343	0.4660
$\rho = -0.2$	$\lambda = 0$	0.4652	0.4640	0.4619	2.2976	0.4684	0.3962	0.3982	0.3979	2.2908	0.4067
$\rho = 0$	$\lambda = 0$	0.4179	0.4163	0.4151	2.2828	0.4162	0.4059	0.4013	0.4007	2.3299	0.4092
$\rho = 0.2$	$\lambda = 0$	0.4020	0.3951	0.3923	2.3080	0.3806	0.4360	0.4308	0.4307	2.3196	0.4261
$\rho = 0.4$	$\lambda = 0$	0.4387	0.4326	0.4274	2.4299	0.4157	0.5156	0.4998	0.4997	2.3908	0.4954
$\rho = 0.6$	$\lambda = 0$	0.4767	0.4709	0.4784	2.4827	0.4631	0.3773	0.3830	0.3820	2.3151	0.3828
$\rho = 0.8$	$\lambda = 0$	0.4012	0.3991	0.3978	2.2996	0.3998	0.4462	0.4419	0.4420	2.2683	0.4426
$\rho = -0.8$	$\lambda = 0.2$	0.4119	0.4075	0.4093	2.2877	0.4154	0.4032	0.3951	0.3961	2.3174	0.3965
$\rho = -0.6$	$\lambda = 0.2$	0.4318	0.4247	0.4291	2.2910	0.4206	0.4173	0.4097	0.4093	2.3914	0.4026
$\rho = -0.4$	$\lambda = 0.2$	0.4658	0.4514	0.4574	2.4181	0.4306	0.4740	0.4648	0.4649	2.4064	0.4592
$\rho = -0.2$	$\lambda = 0.2$	0.4451	0.4343	0.4469	2.4181	0.4142	0.4697	0.4682	0.4698	2.3148	0.4770
$\rho = 0$	$\lambda = 0.2$	0.4451	0.4387	0.4374	2.3771	0.4345	0.4825	0.4795	0.4758	2.2696	0.4750
$\rho = 0.2$	$\lambda = 0.2$	0.4152	0.4087	0.4100	2.2764	0.3921	0.4797	0.4650	0.4795	2.3911	0.4314
$\rho = 0.4$	$\lambda = 0.2$	0.5157	0.5067	0.5015	2.3390	0.5140	0.4444	0.4451	0.4444	2.3159	0.4586
$\rho = 0.6$	$\lambda = 0.2$	0.4698	0.4619	0.4543	2.3788	0.4560	0.4846	0.4701	0.4728	2.4567	0.4605
$\rho = 0.8$	$\lambda = 0.2$	0.4765	0.4720	0.4723	2.4368	0.4713	0.3945	0.3982	0.3934	2.3626	0.4051
$\rho = -0.8$	$\lambda = 0.6$	0.4996	0.4958	0.4939	2.3584	0.5066	0.4478	0.4441	0.4421	2.3226	0.4497
$\rho = -0.6$	$\lambda = 0.6$	0.4191	0.4089	0.4130	2.3536	0.3970	0.4277	0.4243	0.4259	2.3032	0.4303
$\rho = -0.4$	$\lambda = 0.6$	0.4040	0.3985	0.3932	2.3314	0.4158	0.4532	0.4456	0.4443	2.3085	0.4483
$\rho = -0.2$	$\lambda = 0.6$	0.4949	0.4772	0.4868	2.5547	0.4639	0.3941	0.3798	0.3835	2.3285	0.3997
$\rho = 0$	$\lambda = 0.6$	0.4502	0.4568	0.4539	2.4807	0.4556	0.4291	0.4268	0.4249	2.3632	0.4383
$\rho = 0.2$	$\lambda = 0.6$	0.4769	0.4730	0.4715	2.3654	0.4718	0.4436	0.4388	0.4386	2.3375	0.4258
$\rho = 0.4$	$\lambda = 0.6$	0.4365	0.4299	0.4331	2.3196	0.4152	0.4351	0.4291	0.4286	2.3148	0.4139
$\rho = 0.6$	$\lambda = 0.6$	0.4697	0.4631	0.4696	2.3598	0.4363	0.4344	0.4248	0.4292	2.3023	0.4319
$\rho = 0.8$	$\lambda = 0.6$	0.4908	0.4695	0.4809	2.4093	0.4520	0.4823	0.4684	0.4682	2.5202	0.4764
	Average	0.4491	0.4427	0.4446	2.3689	0.4353	0.4428	0.4382	0.4381	2.3419	0.4352

Note: N = 100, T = 5, and K = 2.

Table (4) RMSEs of θ using Mutl & Pfaffermayr (2011), the three sets of GM estimators, and the ML estimator.

		2 ahead and 2 behind					6 ahead and 6 behind				
		IN	FW	W	MP	ML	IN	FW	W	MP	ML
$\rho=-0.8$	$\lambda=-0.6$	0.1077	0.1089	0.1075	0.1749	0.1055	0.1039	0.1048	0.1048	0.1720	0.0977
$\rho=-0.6$	$\lambda=-0.6$	0.1176	0.1184	0.1166	0.1744	0.1186	0.1156	0.1144	0.1157	0.1754	0.1188
$\rho=-0.4$	$\lambda=-0.6$	0.1187	0.1163	0.1206	0.1717	0.1074	0.1224	0.1245	0.1238	0.1766	0.1359
$\rho=-0.2$	$\lambda=-0.6$	0.1162	0.1146	0.1161	0.1716	0.1147	0.1325	0.1330	0.1326	0.1727	0.1272
$\rho=0$	$\lambda=-0.6$	0.1259	0.1242	0.1258	0.1736	0.1209	0.1156	0.1143	0.1142	0.1754	0.1041
$\rho=0.2$	$\lambda=-0.6$	0.1169	0.1193	0.1165	0.1763	0.1132	0.1006	0.0998	0.1007	0.1734	0.1028
$\rho=0.4$	$\lambda=-0.6$	0.1120	0.1095	0.1131	0.1699	0.1038	0.1195	0.1183	0.1180	0.1712	0.1169
$\rho=0.6$	$\lambda=-0.6$	0.1131	0.1125	0.1146	0.1737	0.1129	0.1191	0.1195	0.1195	0.1698	0.1066
$\rho=0.8$	$\lambda=-0.6$	0.1260	0.1240	0.1265	0.1700	0.1013	0.1204	0.1207	0.1200	0.1727	0.1148
$\rho=-0.8$	$\lambda=-0.2$	0.1145	0.1129	0.1140	0.1762	0.1061	0.1206	0.1192	0.1197	0.1734	0.1169
$\rho=-0.6$	$\lambda=-0.2$	0.1015	0.1008	0.1023	0.1758	0.0952	0.1296	0.1283	0.1281	0.1716	0.1312
$\rho=-0.4$	$\lambda=-0.2$	0.1096	0.1097	0.1062	0.1715	0.1103	0.1283	0.1390	0.1371	0.1728	0.1365
$\rho=-0.2$	$\lambda=-0.2$	0.1372	0.1357	0.1343	0.1706	0.1356	0.1124	0.1113	0.1128	0.1701	0.1114
$\rho=0$	$\lambda=-0.2$	0.1294	0.1280	0.1293	0.1768	0.1175	0.1122	0.1123	0.1129	0.1702	0.1115
$\rho=0.2$	$\lambda=-0.2$	0.1206	0.1265	0.1233	0.1758	0.1110	0.1455	0.1443	0.1453	0.1763	0.1322
$\rho=0.4$	$\lambda=-0.2$	0.1028	0.1019	0.1026	0.1768	0.0985	0.1163	0.1159	0.1162	0.1706	0.1187
$\rho=0.6$	$\lambda=-0.2$	0.1029	0.1032	0.0993	0.1702	0.1019	0.1335	0.1329	0.1331	0.1698	0.1321
$\rho=0.8$	$\lambda=-0.2$	0.1209	0.1192	0.1212	0.1698	0.1217	0.1000	0.0992	0.0997	0.1701	0.0983
$\rho=-0.8$	$\lambda=0$	0.1152	0.1150	0.1120	0.1727	0.1101	0.1237	0.1215	0.1232	0.1758	0.1244
$\rho=-0.6$	$\lambda=0$	0.1329	0.1351	0.1310	0.1752	0.1222	0.1216	0.1207	0.1222	0.1721	0.1196
$\rho=-0.4$	$\lambda=0$	0.1348	0.1352	0.1350	0.1774	0.1362	0.1279	0.1338	0.1305	0.1732	0.1393
$\rho=-0.2$	$\lambda=0$	0.1218	0.1230	0.1214	0.1739	0.1240	0.1010	0.0998	0.1011	0.1730	0.0974
$\rho=0$	$\lambda=0$	0.1118	0.1124	0.1123	0.1712	0.1142	0.1047	0.1039	0.1039	0.1741	0.1095
$\rho=0.2$	$\lambda=0$	0.0990	0.1002	0.0979	0.1717	0.0999	0.0992	0.0977	0.0976	0.1724	0.0973
$\rho=0.4$	$\lambda=0$	0.1245	0.1213	0.1227	0.1745	0.1175	0.1384	0.1371	0.1371	0.1758	0.1313
$\rho=0.6$	$\lambda=0$	0.1530	0.1509	0.1551	0.1785	0.1471	0.1063	0.1055	0.1066	0.1731	0.1052
$\rho=0.8$	$\lambda=0$	0.1218	0.1200	0.1206	0.1721	0.1173	0.1159	0.1149	0.1151	0.1716	0.1135
$\rho=-0.8$	$\lambda=0.2$	0.1161	0.1140	0.1159	0.1741	0.1141	0.1190	0.1180	0.1177	0.1728	0.1173
$\rho=-0.6$	$\lambda=0.2$	0.1012	0.0998	0.0995	0.1714	0.0961	0.1147	0.1132	0.1132	0.1754	0.1125
$\rho=-0.4$	$\lambda=0.2$	0.1195	0.1173	0.1200	0.1768	0.1134	0.1328	0.1303	0.1302	0.1737	0.1305
$\rho=-0.2$	$\lambda=0.2$	0.1147	0.1131	0.1127	0.1735	0.1160	0.1163	0.1169	0.1179	0.1736	0.1189
$\rho=0$	$\lambda=0.2$	0.1181	0.1165	0.1176	0.1718	0.1123	0.1482	0.1467	0.1466	0.1701	0.1452
$\rho=0.2$	$\lambda=0.2$	0.1026	0.1011	0.1043	0.1702	0.0980	0.1186	0.1148	0.1197	0.1781	0.1109
$\rho=0.4$	$\lambda=0.2$	0.1389	0.1381	0.1359	0.1726	0.1398	0.1027	0.1025	0.1028	0.1709	0.1050
$\rho=0.6$	$\lambda=0.2$	0.1269	0.1259	0.1231	0.1745	0.1257	0.1359	0.1339	0.1346	0.1786	0.1288
$\rho=0.8$	$\lambda=0.2$	0.1237	0.1234	0.1237	0.1733	0.1130	0.1257	0.1252	0.1252	0.1715	0.1233
$\rho=-0.8$	$\lambda=0.6$	0.1341	0.1319	0.1333	0.1740	0.1313	0.1226	0.1220	0.1219	0.1691	0.1240
$\rho=-0.6$	$\lambda=0.6$	0.1095	0.1088	0.1098	0.1707	0.1092	0.1055	0.1042	0.1069	0.1725	0.1063
$\rho=-0.4$	$\lambda=0.6$	0.0991	0.0974	0.0979	0.1729	0.1023	0.1223	0.1208	0.1215	0.1750	0.1208
$\rho=-0.2$	$\lambda=0.6$	0.1393	0.1407	0.1418	0.1780	0.1342	0.0987	0.0969	0.0975	0.1709	0.1068
$\rho=0$	$\lambda=0.6$	0.1266	0.1269	0.1281	0.1745	0.1137	0.1081	0.1085	0.1076	0.1727	0.1068
$\rho=0.2$	$\lambda=0.6$	0.1382	0.1424	0.1396	0.1735	0.1386	0.1149	0.1142	0.1144	0.1739	0.1075
$\rho=0.4$	$\lambda=0.6$	0.1191	0.1179	0.1189	0.1713	0.1105	0.1079	0.1064	0.1066	0.1705	0.1022
$\rho=0.6$	$\lambda=0.6$	0.1322	0.1311	0.1335	0.1715	0.1192	0.1228	0.1214	0.1224	0.1706	0.1102
$\rho=0.8$	$\lambda=0.6$	0.1389	0.1359	0.1419	0.1757	0.1256	0.1269	0.1240	0.1239	0.1814	0.1289
	Average	0.1202	0.1196	0.1199	0.1735	0.1155	0.1184	0.1179	0.1183	0.1730	0.1168

Note: N = 100, T = 5, and K = 2.

Table (5) RMSEs of λ using Mutl & Pfaffermayr (2011), the three sets of GM estimators, and the ML estimator.

		2 ahead and 2 behind					6 ahead and 6 behind				
		IN	FW	W	MP	ML	IN	FW	W	MP	ML
$\rho=-0.8$	$\lambda=-0.6$	0.2357	0.2341	0.2352	0.2661	0.1886	0.4341	0.4379	0.4364	0.4322	0.2192
$\rho=-0.6$	$\lambda=-0.6$	0.2124	0.2139	0.2138	0.2514	0.1834	0.4616	0.4589	0.4597	0.5634	0.2292
$\rho=-0.4$	$\lambda=-0.6$	0.2068	0.2099	0.2082	0.2554	0.1777	0.5256	0.5184	0.5204	0.5422	0.2286
$\rho=-0.2$	$\lambda=-0.6$	0.2558	0.2574	0.2568	0.2366	0.1816	0.6545	0.6563	0.6539	0.7061	0.2355
$\rho=0$	$\lambda=-0.6$	0.2055	0.2017	0.2033	0.2499	0.1859	0.6208	0.6078	0.6079	0.7278	0.2447
$\rho=0.2$	$\lambda=-0.6$	0.2248	0.2174	0.2190	0.2542	0.2370	0.4148	0.4213	0.4112	0.4685	0.3526
$\rho=0.4$	$\lambda=-0.6$	0.2604	0.2595	0.2598	0.2779	0.2461	0.5114	0.5088	0.5084	0.5348	0.3740
$\rho=0.6$	$\lambda=-0.6$	0.2607	0.2654	0.2637	0.2564	0.2183	0.5946	0.5907	0.5961	0.6815	0.3784
$\rho=0.8$	$\lambda=-0.6$	0.2110	0.2047	0.2053	0.2552	0.1916	0.5336	0.5336	0.5322	0.6019	0.3824
$\rho=-0.8$	$\lambda=-0.2$	0.2588	0.2668	0.2649	0.2652	0.2261	0.6311	0.6296	0.6279	0.7666	0.3987
$\rho=-0.6$	$\lambda=-0.2$	0.2313	0.2309	0.2305	0.2352	0.2327	0.4389	0.4377	0.4382	0.4715	0.4187
$\rho=-0.4$	$\lambda=-0.2$	0.2785	0.2790	0.2790	0.2899	0.2692	0.5831	0.5823	0.5830	0.6141	0.5113
$\rho=-0.2$	$\lambda=-0.2$	0.2136	0.2078	0.2076	0.2610	0.2037	0.5186	0.5262	0.5163	0.7146	0.5296
$\rho=0$	$\lambda=-0.2$	0.3160	0.3199	0.3202	0.2687	0.2513	0.5905	0.5884	0.5902	0.7117	0.5130
$\rho=0.2$	$\lambda=-0.2$	0.2989	0.2996	0.3005	0.3267	0.2425	0.6683	0.6683	0.6643	0.6769	0.5490
$\rho=0.4$	$\lambda=-0.2$	0.2263	0.2338	0.2304	0.2288	0.2204	0.5187	0.5245	0.5209	0.5455	0.4633
$\rho=0.6$	$\lambda=-0.2$	0.2163	0.2165	0.2161	0.2537	0.2215	0.4929	0.4838	0.4900	0.4710	0.4550
$\rho=0.8$	$\lambda=-0.2$	0.2470	0.2498	0.2487	0.2392	0.2322	0.4976	0.4960	0.4969	0.5996	0.5133
$\rho=-0.8$	$\lambda=0$	0.2645	0.2626	0.2620	0.3070	0.2667	0.5977	0.5995	0.5949	0.6089	0.4920
$\rho=-0.6$	$\lambda=0$	0.3468	0.3396	0.3410	0.4301	0.2590	0.7060	0.7004	0.7017	0.7662	0.4762
$\rho=-0.4$	$\lambda=0$	0.2072	0.2070	0.2070	0.2393	0.1808	0.3332	0.3250	0.3268	0.4084	0.3413
$\rho=-0.2$	$\lambda=0$	0.2240	0.2224	0.2220	0.2519	0.2116	0.4850	0.4849	0.4857	0.5372	0.4302
$\rho=0$	$\lambda=0$	0.2605	0.2572	0.2568	0.2894	0.2413	0.4743	0.4712	0.4729	0.6257	0.4410
$\rho=0.2$	$\lambda=0$	0.3231	0.3215	0.3223	0.2982	0.2380	0.5198	0.5210	0.5207	0.5740	0.6188
$\rho=0.4$	$\lambda=0$	0.3442	0.3393	0.3415	0.3670	0.2581	0.6656	0.6629	0.6650	0.7053	0.5682
$\rho=0.6$	$\lambda=0$	0.2350	0.2356	0.2355	0.2480	0.2254	0.3290	0.3172	0.3212	0.3736	0.3131
$\rho=0.8$	$\lambda=0$	0.1984	0.1976	0.1970	0.2193	0.1874	0.3946	0.3872	0.3859	0.4642	0.4503
$\rho=-0.8$	$\lambda=0.2$	0.2228	0.2222	0.2228	0.2510	0.2034	0.4295	0.4260	0.4261	0.4733	0.4611
$\rho=-0.6$	$\lambda=0.2$	0.2522	0.2523	0.2525	0.2689	0.2521	0.5040	0.5039	0.5035	0.6131	0.4716
$\rho=-0.4$	$\lambda=0.2$	0.3368	0.3328	0.3348	0.3776	0.2796	0.9184	0.9400	0.9403	0.8911	0.6918
$\rho=-0.2$	$\lambda=0.2$	0.1560	0.1542	0.1546	0.1931	0.1408	0.3149	0.3115	0.3129	0.3583	0.2732
$\rho=0$	$\lambda=0.2$	0.1982	0.1976	0.1978	0.2040	0.1832	0.3819	0.3762	0.3753	0.4235	0.3757
$\rho=0.2$	$\lambda=0.2$	0.1927	0.1959	0.1959	0.2153	0.1907	0.3590	0.3540	0.3525	0.4547	0.4327
$\rho=0.4$	$\lambda=0.2$	0.2430	0.2426	0.2427	0.2730	0.2092	0.4259	0.4139	0.4198	0.5450	0.4028
$\rho=0.6$	$\lambda=0.2$	0.3134	0.3112	0.3111	0.3670	0.2767	0.5567	0.5737	0.5727	0.7313	0.4678
$\rho=0.8$	$\lambda=0.2$	0.1029	0.1016	0.1025	0.1217	0.0982	0.1773	0.1775	0.1769	0.1953	0.1789
$\rho=-0.8$	$\lambda=0.6$	0.1465	0.1465	0.1463	0.1556	0.1420	0.2812	0.2778	0.2774	0.3109	0.2888
$\rho=-0.6$	$\lambda=0.6$	0.1896	0.1878	0.1875	0.1991	0.1834	0.3604	0.3554	0.3578	0.4257	0.4286
$\rho=-0.4$	$\lambda=0.6$	0.2015	0.1919	0.1924	0.2849	0.1914	0.4297	0.4261	0.4264	0.4845	0.4330
$\rho=-0.2$	$\lambda=0.6$	0.2871	0.2974	0.2966	0.3905	0.2713	0.5449	0.5393	0.5432	0.6365	0.4958
$\rho=0$	$\lambda=0.6$	0.0707	0.0706	0.0708	0.0778	0.0599	0.1277	0.1277	0.1276	0.1475	0.1182
$\rho=0.2$	$\lambda=0.6$	0.0941	0.0942	0.0942	0.0989	0.0815	0.1632	0.1501	0.1571	0.3194	0.1469
$\rho=0.4$	$\lambda=0.6$	0.0909	0.0909	0.0910	0.1014	0.0739	0.2081	0.2094	0.2123	0.2597	0.2520
$\rho=0.6$	$\lambda=0.6$	0.1256	0.1254	0.1255	0.1406	0.1220	0.4197	0.4141	0.4136	0.5213	0.6005
$\rho=0.8$	$\lambda=0.6$	0.1790	0.2187	0.1888	0.2721	0.2522	0.5125	0.5150	0.5202	0.5328	0.4006
	Average	0.2259	0.2263	0.2257	0.2514	0.2042	0.4736	0.4718	0.4721	0.5382	0.4011

Note: N = 100, T = 5, and K = 2.

Table (6) RMSEs of β_1 using Mutl & Pfaffermayr (2011), the three sets of GM estimators, and the ML estimator.

		2 ahead and 2 behind					6 ahead and 6 behind				
		IN	FW	W	MP	ML	IN	FW	W	MP	ML
$\rho = -0.8$	$\lambda = -0.6$	0.1130	0.1145	0.1139	0.1248	0.1168	0.1255	0.1253	0.1255	0.1366	0.1264
$\rho = -0.6$	$\lambda = -0.6$	0.1191	0.1188	0.1188	0.1367	0.1163	0.1191	0.1184	0.1182	0.1334	0.1159
$\rho = -0.4$	$\lambda = -0.6$	0.1157	0.1150	0.1149	0.1242	0.1057	0.1341	0.1341	0.1336	0.1333	0.1357
$\rho = -0.2$	$\lambda = -0.6$	0.1578	0.1578	0.1578	0.1708	0.1462	0.1435	0.1457	0.1457	0.1291	0.1085
$\rho = 0$	$\lambda = -0.6$	0.1508	0.1483	0.1486	0.1517	0.1290	0.1809	0.1793	0.1796	0.1820	0.1356
$\rho = 0.2$	$\lambda = -0.6$	0.1159	0.1154	0.1158	0.1353	0.1152	0.1261	0.1261	0.1260	0.1401	0.1262
$\rho = 0.4$	$\lambda = -0.6$	0.1159	0.1149	0.1150	0.1351	0.1151	0.1291	0.1291	0.1291	0.1544	0.1246
$\rho = 0.6$	$\lambda = -0.6$	0.1459	0.1444	0.1447	0.1599	0.1451	0.1333	0.1323	0.1325	0.1507	0.1316
$\rho = 0.8$	$\lambda = -0.6$	0.1319	0.1307	0.1308	0.1605	0.1234	0.1461	0.1466	0.1472	0.1391	0.1359
$\rho = -0.8$	$\lambda = -0.2$	0.1750	0.1750	0.1750	0.1931	0.1537	0.1493	0.1482	0.1482	0.1565	0.1547
$\rho = -0.6$	$\lambda = -0.2$	0.1159	0.1151	0.1150	0.1350	0.1148	0.1184	0.1192	0.1191	0.1322	0.1126
$\rho = -0.4$	$\lambda = -0.2$	0.1150	0.1141	0.1140	0.1430	0.1139	0.1099	0.1090	0.1097	0.1311	0.1149
$\rho = -0.2$	$\lambda = -0.2$	0.1371	0.1356	0.1359	0.1303	0.1355	0.1302	0.1302	0.1301	0.1573	0.1257
$\rho = 0$	$\lambda = -0.2$	0.1479	0.1473	0.1472	0.1569	0.1460	0.1194	0.1182	0.1182	0.1476	0.1204
$\rho = 0.2$	$\lambda = -0.2$	0.1430	0.1434	0.1431	0.1561	0.1300	0.1473	0.1458	0.1463	0.1685	0.1451
$\rho = 0.4$	$\lambda = -0.2$	0.1284	0.1297	0.1280	0.1415	0.1145	0.1266	0.1277	0.1278	0.1538	0.1267
$\rho = 0.6$	$\lambda = -0.2$	0.1121	0.1122	0.1122	0.1239	0.1128	0.1322	0.1313	0.1316	0.1418	0.1370
$\rho = 0.8$	$\lambda = -0.2$	0.1326	0.1327	0.1327	0.1473	0.1324	0.1229	0.1234	0.1233	0.1423	0.1217
$\rho = -0.8$	$\lambda = 0$	0.1213	0.1221	0.1221	0.1225	0.1151	0.1255	0.1258	0.1255	0.1430	0.1231
$\rho = -0.6$	$\lambda = 0$	0.1698	0.1701	0.1700	0.1878	0.1332	0.1708	0.1697	0.1699	0.1693	0.1453
$\rho = -0.4$	$\lambda = 0$	0.1333	0.1320	0.1323	0.1475	0.1181	0.1311	0.1304	0.1311	0.1321	0.1292
$\rho = -0.2$	$\lambda = 0$	0.1215	0.1216	0.1213	0.1388	0.1207	0.1287	0.1287	0.1287	0.1391	0.1284
$\rho = 0$	$\lambda = 0$	0.1152	0.1154	0.1154	0.1151	0.1142	0.1217	0.1217	0.1217	0.1299	0.1257
$\rho = 0.2$	$\lambda = 0$	0.1547	0.1535	0.1539	0.1396	0.1357	0.1241	0.1230	0.1231	0.1392	0.1280
$\rho = 0.4$	$\lambda = 0$	0.1409	0.1404	0.1402	0.1742	0.1221	0.1380	0.1381	0.1381	0.1348	0.1347
$\rho = 0.6$	$\lambda = 0$	0.1528	0.1482	0.1501	0.1723	0.1441	0.1324	0.1326	0.1324	0.1421	0.1324
$\rho = 0.8$	$\lambda = 0$	0.1270	0.1271	0.1267	0.1560	0.1221	0.1361	0.1361	0.1361	0.1498	0.1354
$\rho = -0.8$	$\lambda = 0.2$	0.1117	0.1107	0.1117	0.1263	0.1090	0.1460	0.1459	0.1459	0.1360	0.1459
$\rho = -0.6$	$\lambda = 0.2$	0.1148	0.1154	0.1156	0.1381	0.1151	0.1379	0.1380	0.1380	0.1456	0.1407
$\rho = -0.4$	$\lambda = 0.2$	0.1092	0.1092	0.1099	0.1217	0.1284	0.1504	0.1485	0.1486	0.1563	0.1323
$\rho = -0.2$	$\lambda = 0.2$	0.1373	0.1374	0.1374	0.1527	0.1291	0.1389	0.1393	0.1396	0.1415	0.1416
$\rho = 0$	$\lambda = 0.2$	0.1227	0.1239	0.1235	0.1238	0.1267	0.1190	0.1185	0.1185	0.1247	0.1201
$\rho = 0.2$	$\lambda = 0.2$	0.1306	0.1299	0.1299	0.1460	0.1328	0.1325	0.1325	0.1325	0.1325	0.1311
$\rho = 0.4$	$\lambda = 0.2$	0.1492	0.1480	0.1480	0.1581	0.1491	0.1236	0.1236	0.1235	0.1476	0.1229
$\rho = 0.6$	$\lambda = 0.2$	0.1356	0.1337	0.1341	0.1435	0.1382	0.1198	0.1179	0.1176	0.1289	0.1294
$\rho = 0.8$	$\lambda = 0.2$	0.1394	0.1390	0.1405	0.1387	0.1258	0.1211	0.1220	0.1214	0.1313	0.1293
$\rho = -0.8$	$\lambda = 0.6$	0.1414	0.1414	0.1418	0.1509	0.1341	0.1284	0.1288	0.1286	0.1405	0.1290
$\rho = -0.6$	$\lambda = 0.6$	0.1321	0.1309	0.1316	0.1342	0.1227	0.1251	0.1247	0.1255	0.1328	0.1225
$\rho = -0.4$	$\lambda = 0.6$	0.1301	0.1301	0.1301	0.1314	0.1308	0.1224	0.1224	0.1224	0.1301	0.1224
$\rho = -0.2$	$\lambda = 0.6$	0.1342	0.1346	0.1346	0.1359	0.1364	0.1231	0.1236	0.1236	0.1319	0.1255
$\rho = 0$	$\lambda = 0.6$	0.1290	0.1294	0.1293	0.1452	0.1341	0.1349	0.1348	0.1347	0.1805	0.1310
$\rho = 0.2$	$\lambda = 0.6$	0.1382	0.1416	0.1394	0.1393	0.1333	0.1233	0.1243	0.1241	0.1307	0.1235
$\rho = 0.4$	$\lambda = 0.6$	0.1646	0.1646	0.1644	0.1733	0.1693	0.1154	0.1154	0.1154	0.1197	0.1151
$\rho = 0.6$	$\lambda = 0.6$	0.1338	0.1335	0.1333	0.1270	0.1266	0.1230	0.1232	0.1230	0.1256	0.1467
$\rho = 0.8$	$\lambda = 0.6$	0.1512	0.1283	0.1281	0.1493	0.1268	0.1166	0.1166	0.1166	0.1265	0.1152
	Average	0.1337	0.1328	0.1329	0.1448	0.1280	0.1312	0.1310	0.1311	0.1416	0.1290

Note: N = 100, T = 5, and K = 2.

Table (7) RMSEs of β_2 using Mutl & Pfaffermayr (2011), the three sets of GM estimators, and the ML estimator.

		2 ahead and 2 behind					6 ahead and 6 behind				
		IN	FW	W	MP	ML	IN	FW	W	MP	ML
$\rho = -0.8$	$\lambda = -0.6$	0.1834	0.1823	0.1831	0.1862	0.1809	0.2175	0.2185	0.2188	0.2195	0.2198
$\rho = -0.6$	$\lambda = -0.6$	0.2284	0.2218	0.2230	0.2303	0.2206	0.2260	0.2256	0.2262	0.2221	0.2259
$\rho = -0.4$	$\lambda = -0.6$	0.2000	0.2003	0.1993	0.2021	0.1965	0.2291	0.2313	0.2307	0.2354	0.2296
$\rho = -0.2$	$\lambda = -0.6$	0.2179	0.2164	0.2164	0.2167	0.2150	0.2159	0.2159	0.2158	0.2119	0.2161
$\rho = 0$	$\lambda = -0.6$	0.2175	0.2187	0.2181	0.2240	0.2091	0.2272	0.2315	0.2308	0.2191	0.2281
$\rho = 0.2$	$\lambda = -0.6$	0.1925	0.1896	0.1922	0.1860	0.1879	0.1959	0.1960	0.1954	0.1987	0.1955
$\rho = 0.4$	$\lambda = -0.6$	0.1973	0.1988	0.1986	0.1960	0.1994	0.1841	0.1835	0.1831	0.1813	0.1844
$\rho = 0.6$	$\lambda = -0.6$	0.2368	0.2362	0.2356	0.2355	0.2344	0.2306	0.2320	0.2310	0.2308	0.2328
$\rho = 0.8$	$\lambda = -0.6$	0.2145	0.2147	0.2154	0.2150	0.2183	0.2280	0.2302	0.2295	0.2274	0.2180
$\rho = -0.8$	$\lambda = -0.2$	0.2338	0.2292	0.2304	0.2376	0.2093	0.1997	0.2030	0.2029	0.1950	0.1955
$\rho = -0.6$	$\lambda = -0.2$	0.1990	0.1994	0.1990	0.1961	0.1949	0.2152	0.2177	0.2162	0.2144	0.2186
$\rho = -0.4$	$\lambda = -0.2$	0.2422	0.2435	0.2432	0.2419	0.2431	0.2174	0.2191	0.2186	0.2175	0.2267
$\rho = -0.2$	$\lambda = -0.2$	0.1940	0.1962	0.1963	0.1937	0.1953	0.2246	0.2255	0.2250	0.2257	0.2253
$\rho = 0$	$\lambda = -0.2$	0.2203	0.2213	0.2213	0.2197	0.2237	0.1961	0.1985	0.1974	0.1869	0.2032
$\rho = 0.2$	$\lambda = -0.2$	0.2189	0.2194	0.2196	0.2268	0.2001	0.2429	0.2410	0.2425	0.2527	0.2405
$\rho = 0.4$	$\lambda = -0.2$	0.2399	0.2410	0.2413	0.2398	0.2390	0.2123	0.2115	0.2114	0.2096	0.2053
$\rho = 0.6$	$\lambda = -0.2$	0.2199	0.2220	0.2210	0.2216	0.2238	0.2395	0.2402	0.2405	0.2367	0.2376
$\rho = 0.8$	$\lambda = -0.2$	0.2316	0.2242	0.2231	0.2308	0.2221	0.2454	0.2472	0.2468	0.2446	0.2493
$\rho = -0.8$	$\lambda = 0$	0.2370	0.2369	0.2370	0.2360	0.2380	0.2428	0.2379	0.2380	0.2438	0.2419
$\rho = -0.6$	$\lambda = 0$	0.2236	0.2164	0.2164	0.2182	0.2247	0.2136	0.2138	0.2139	0.2092	0.2040
$\rho = -0.4$	$\lambda = 0$	0.1951	0.1979	0.1965	0.1982	0.2045	0.2539	0.2553	0.2546	0.2530	0.2556
$\rho = -0.2$	$\lambda = 0$	0.2112	0.2133	0.2131	0.2115	0.2164	0.2101	0.2098	0.2099	0.2097	0.2126
$\rho = 0$	$\lambda = 0$	0.2446	0.2367	0.2373	0.2465	0.2379	0.2541	0.2532	0.2539	0.2516	0.2533
$\rho = 0.2$	$\lambda = 0$	0.2182	0.2194	0.2194	0.2172	0.2191	0.2198	0.2187	0.2195	0.2176	0.2166
$\rho = 0.4$	$\lambda = 0$	0.2495	0.2492	0.2490	0.2447	0.2394	0.2404	0.2422	0.2425	0.2404	0.2459
$\rho = 0.6$	$\lambda = 0$	0.2140	0.2110	0.2103	0.2101	0.2105	0.2332	0.2332	0.2332	0.2359	0.2347
$\rho = 0.8$	$\lambda = 0$	0.2020	0.2023	0.2029	0.2034	0.2029	0.2440	0.2377	0.2460	0.2449	0.2350
$\rho = -0.8$	$\lambda = 0.2$	0.2000	0.1980	0.1983	0.1969	0.1964	0.2315	0.2331	0.2323	0.2304	0.2360
$\rho = -0.6$	$\lambda = 0.2$	0.2122	0.2100	0.2095	0.2091	0.2067	0.2739	0.2711	0.2744	0.2730	0.2714
$\rho = -0.4$	$\lambda = 0.2$	0.2813	0.2856	0.2852	0.2784	0.2873	0.2684	0.2713	0.2721	0.2601	0.2442
$\rho = -0.2$	$\lambda = 0.2$	0.2097	0.2048	0.2071	0.2177	0.1950	0.2070	0.2081	0.2061	0.2125	0.2154
$\rho = 0$	$\lambda = 0.2$	0.2032	0.2029	0.2036	0.2091	0.2053	0.2514	0.2420	0.2423	0.2541	0.2387
$\rho = 0.2$	$\lambda = 0.2$	0.2278	0.2103	0.2126	0.2297	0.2145	0.2410	0.2414	0.2419	0.2408	0.2431
$\rho = 0.4$	$\lambda = 0.2$	0.2117	0.2126	0.2124	0.2122	0.2136	0.2545	0.2558	0.2556	0.2532	0.2562
$\rho = 0.6$	$\lambda = 0.2$	0.2051	0.2058	0.2050	0.2059	0.2073	0.2693	0.2685	0.2690	0.2681	0.2647
$\rho = 0.8$	$\lambda = 0.2$	0.2512	0.2516	0.2508	0.2481	0.2479	0.2265	0.2256	0.2259	0.2247	0.2305
$\rho = -0.8$	$\lambda = 0.6$	0.2197	0.2180	0.2176	0.2221	0.2139	0.2175	0.2186	0.2181	0.2164	0.2152
$\rho = -0.6$	$\lambda = 0.6$	0.2034	0.2040	0.2044	0.2067	0.2067	0.2065	0.2031	0.2049	0.2100	0.1999
$\rho = -0.4$	$\lambda = 0.6$	0.1970	0.1887	0.1887	0.1974	0.1910	0.2557	0.2025	0.2203	0.2565	0.2043
$\rho = -0.2$	$\lambda = 0.6$	0.2330	0.2345	0.2345	0.2343	0.2371	0.1857	0.1859	0.1874	0.1836	0.1941
$\rho = 0$	$\lambda = 0.6$	0.2387	0.2401	0.2394	0.2414	0.2437	0.2233	0.2247	0.2240	0.2259	0.2302
$\rho = 0.2$	$\lambda = 0.6$	0.2225	0.2221	0.2219	0.2116	0.2079	0.2210	0.2209	0.2196	0.2204	0.2162
$\rho = 0.4$	$\lambda = 0.6$	0.2460	0.2475	0.2468	0.2490	0.2523	0.2343	0.2359	0.2359	0.2313	0.2354
$\rho = 0.6$	$\lambda = 0.6$	0.2156	0.2164	0.2173	0.2181	0.2120	0.2227	0.2238	0.2239	0.2219	0.2258
$\rho = 0.8$	$\lambda = 0.6$	0.1898	0.1882	0.1884	0.1889	0.1889	0.2538	0.2444	0.2472	0.2540	0.2431
	Average	0.2189	0.2178	0.2178	0.2192	0.2163	0.2290	0.2277	0.2283	0.2283	0.2270

Note: N = 100, T = 5, and K = 2.