Architecture-Based Software Reliability: Why Only a Few Parameters Matter?

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Abstract

Uncertainty analysis through sensitivity studies and quantification of the variance of the reliability estimate has become more common in architecture-based software reliability studies. However, up to this point no attempts have been made to explicate the results of such analysis. Our earlier work based on several medium to large scale empirical studies showed that a very few parameters have a significant impact on the variability of system reliability. This paper explains the reasons behind this phenomenon. Unlike related work that considered the impact of the parameters on software reliability either through their model sensitivity or through uncertainty of their estimates, we consider both. Furthermore, we look at all parameters, i.e., components reliabilities and probabilities of transfer of control between components. Based on theoretical and empirical arguments, we justify why a few parameters contribute most of the variance of the reliability estimate. Comparing our results with those obtained through simple model sensitivity studies shows that such studies are not always sufficient to accurately quantify the impact of critical components on variability of system reliability.

1. Introduction

Architecture–based software reliability models provide reliability estimates that take into account components failure behavior (expressed for example by components reliabilities R_i) and the way these components interact (usually expressed through probabilities p_{ij} of control transfer from component i to component j). There are many open questions surrounding the estimates of software reliability, especially when concerning the parameters used as input to the models. Parameters can be estimated based on field data collected during testing and/or operational usage, historical data from similar software, or the specifications and design documentation. In practice, there is a lot of uncertainty around parameters because they rarely can be estimated accurately. Although, uncertainty analysis through sensitivity studies and quantification of the variability of reliability has become more common in software reliability, up to this point no attempts have been made to explicate the results of such analysis.

In [8], [9], and [11] we presented uncertainty analysis based on the method of moments and Monte Carlo simulations using several medium to large scale real case studies. As a part of the Monte Carlo simulations, the contribution of each parameter to the variability of the system reliability estimates was measured by estimating the Pearson's correlation coefficients between the ranks of the sampled values of that parameter and the ranks of the corresponding estimated values of the system reliability. The results showed that very few parameters are responsible for most of the variability of the system reliability estimates. In particular, in the study of the European Space Agency (ESA) software, which consists of about 10,000 lines of code, *2 out of 6 parameters were responsible for 93.2% of the variability in the reliability estimate* [8]. Similar trend was noticed for the open source application Indent which has about 11,000 lines of code [11]; *4 out of 43 parameters in the reliability model were responsible for 76.4% of the variance in the system reliability estimate. Even more, the top 10 parameters (out of 43) have contributed 99.6% of the variance in the system reliability estimate.*

The work presented in this paper is motivated by these observations. Our goal is to explain, based on theoretical and empirical analysis, why only a few parameters contribute to the most of the variability in the system reliability estimate. The rest of the paper is organized as follows. Section 2 discusses the related work and our contributions. In Section 3 we provide the theoretical approach used in this paper. Descriptions of the case studies and the values of the considered measures are presented in section 4. The summary of the empirical results that support the theoretical arguments and provide explanation of the observed phenomena is given in section 5. Finally, the concluding remarks are given in section 6.

2. Related work and our contributions

An extensive survey of architecture-based software reliability models was presented in [6]. Although numerous papers were devoted to such models, only a few have actually applied the models on real case studies [13], [3], [8], [9], [10], [11], and even fewer have conducted detailed uncertainty analysis on real data (for example see [8], [9], [11]).

Typically, studies of the model sensitivity to the parameters in the context of architecture–based models are conducted by measuring the change in the overall reliability as a single component reliability varies. Thus, in [2] and [16] the authors assumed fixed known values for the transition probabilities and derived the sensitivity of the system reliability with respect to the reliability of each component. The results were illustrated on simple made-up models. The fact that any inac-

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curacy in the operational profile directly affects the transition probabilities between components was not considered in these papers. Several architecture–based software reliability models were compared theoretically in [7]. In addition to the empirical comparison and validation of the models, the sensitivity study with respect to the operational profile (i.e. transition probabilities) and component reliabilities was conducted on a real application which consists of 10,000 lines of code. The work presented in [4] studied the sensitivity of the reliability of a software application to changes in components reliabilities and transition probabilities. The results were illustrated on the same made-up example from [2] and included sensitivity study of two component reliabilities and two transition probabilities chosen arbitrarily.

Another approach to uncertainty analysis is to study how the uncertainty in the estimates of parameters affects the variability in the reliability estimate. In [14] the authors used a Bayesian approach to estimate the moments of the failure probability for software that has not yet exhibited any failures. The work presented in [1] also considered software that has not failed; the input domain was partitioned and it was recognized that the uncertainty also exists in the probability of using each partition. In [15], the mean and variance of software failure probability were estimated using simulation and assuming Beta prior distributions for components failure probabilities. The analytical expressions for the mean and variance of the application reliability based on the hierarchical architecture–based software reliability model were derived in [5] and the results were illustrated on the example from [2]. Neither [15] nor [5] considered the uncertainties in the estimates of transition probabilities.

A methodology for uncertainty analysis of architecture-based software reliability models suitable for large complex component-based system was presented in [8]. Within this methodology, we proposed several methods such as entropy, Monte Carlo simulations, and methods of moments [12], [8], and [9]. These methods were illustrated on several medium to large scale case studies, including a case study from the European Space Agency which consists of about 10,000 lines of code [8], and two open source applications, Indent which consists of about 11,000 lines of code and GNU GCC C compiler which consists of over 300,000 lines of code [11]. One of the main observations that holds for all three case studies was that a very few parameters have a significant impact on the variability of system reliability. However, the reasons behind this phenomenon were not explored in these papers.

Unlike most related studies that considered the impact of the parameters on architecture–based software reliability either through their model sensitivity or through their uncertainty, in this paper we consider both. This is important since the overall sensitivity of the reliability estimate is a combination of these two factors. Thus, as it is shown in this paper, considering only one factor is not sufficient to explain and accurately quantify the impact of some parameters on the variability of the system reliability. Furthermore, we consider the effect of all parameters (i.e., component reliabilities and probabilities of the transfer of control between components) in a systematic way, rather than arbitrarily choosing a few parameters to illustrate the concept.

Our main goal is to explain based on theoretical arguments and empirical data from real case studies the phenomena we observed in [8], [9], [11]: *(1) small number of parameters contribute to the most of the variation in system reliability and (2) given an operational profile, components' reliabilities have more significant impact on system reliability than transition probabilities.* It should be emphasized that although in this paper we use the architecture–based software reliability model first proposed in [2], our main results are valid for any software reliability model based on the same assumptions as [2].

3. Theoretical approach

The architecture–based software reliability model presented in [2] represents the software executions with a discrete time Markov chain (DTMC) where states represent active components and arcs represent the transfer of control between components. Two absorbing states, *end* and F, are added to the DTMC to represent the end of the execution with correct output and failure, respectively. Assuming that components fail independently and that each component failure leads to a system failure, the transition probability matrix $P = [p_{ij}]$ is modified to $\hat{P}$ by multiplying each transition probability p_{ij} by the corresponding component reliability R_i. This represents the probability that component i produces the correct output and transfers control to component j. Then, from the final state n a directed edge with transition probability R_n is drawn to the *end* state, representing correct execution of the entire program. It is assumed that each component fails independently and the failure probability is represented by drawing a directed edge from i to F with a transition probability of $1 - R_i$. The system reliability is then equal to the probability of reaching state *end* in the absorbing DTMC. Let $\hat{Q}$ be the matrix obtained by removing the rows and columns corresponding to states *end* and F from the edited transition probability matrix $\hat{P}$. $\hat{Q}^k_{1,n}$ represents the probability of reaching state n from 1 through k transitions. From initial state 1 to final state n, the number of transitions k may vary from 0 to ∞. It can be shown that $S = \sum_{k=0}^{\infty} \hat{Q}^k = (I - \hat{Q})^{-1}$, so it follows that the overall system reliability is $R = S_{1,n} R_n$ where $S_{1,n}$ is the $(1, n)$ element of the matrix S. It should be noted that the closed form expression for the system reliability is a function of transition probabilities and components reliabilities $R = f(p_{ij}, R_i)$.

Regardless of the model used, the uncertainty involved with the values of the corresponding parameters affects the estimate of the system reliability. Therefore, the way to deal with the uncertainty of the parameters is presented next.

3.1. Uncertainty of the parameters

The point estimate of the reliability of component i is obtained using $R_i = 1 - f_i/n_i$ where f_i is the number of failures of component i, and n_i is the number of executions of component i. Using point estimates does not account for the uncertainty. In addition, it would mean that any component that does not fail during testing has a reliability equal

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to one. Unless exhaustive testing without replacement has been conducted, we cannot claim reliability to be equal to one. Therefore, to more appropriately estimate the component reliabilities of failure-free executions and account for the uncertainty, we follow the Bayesian framework [14]. The number of successes r_i in n_i executions, given component reliability $R_i (0 \leq R_i \leq 1)$, follows the binomial distribution

$$\binom{n_i}{r_i} R_i^{r_i}(1 - R_i)^{n_i - r_i}. \tag{1}$$

Within the Bayesian framework a priori knowledge about the parameter of interest, here R_i, is represented by the prior distribution. In this case we use as a prior distribution the conjugate distribution Beta(a_i, b_i) given with equation

$$f(R_i) = \frac{\Gamma(a_i + b_i)}{\Gamma(a_i)\Gamma(b_i)} R_i^{a_i - 1}(1 - R_i)^{b_i - 1} \tag{2}$$

where $a_i > 0$ and $b_i > 0$. We concentrate on the case when no prior information is available and use the "ignorance" uniform prior Beta(1, 1) in which case the posterior distribution reduces to Beta($r_i + 1, n_i - r_i + 1$).

The point estimates of the transition probabilities are obtained using $p_{ij} = n_{ij}/n_i$ where n_{ij} is the number of times control transfers from component i to component j and $n_i = \sum_j n_{ij}$. Obviously not all operational profiles execute all possible transitions. We use a static code analysis tool to determine which transitions are not possible (i.e., the transition count will always be zero $n_{ij} = 0$) and therefore $p_{ij} = 0$. When the static code analysis shows that the transition is possible, but no transitions were observed during specific number of executions, the transition probability is likely to be close to 0, but is not improbable. To account for this case, as in the case of components reliability when no failures were observed, the Bayesian framework is used [11]. Let r_{ij} denote the number of times the control was passed from component i to component j in n_i executions. Then, the data follows the multinomial distribution

$$\binom{n_i}{r_{i1} r_{i2} \ldots r_{in}} p_{i1}^{r_{i1}} p_{i2}^{r_{i2}} \ldots p_{in}^{r_{in}} \tag{3}$$

for $r_{ij} = 0, 1, 2, \ldots, n_i$ and $\sum_{j=1}^{n} r_{ij} = n_i$. The number of categories in the multinomial distribution will typically be less than n because, as described earlier, some of the transitions are improbable. It is assumed that the rows in the transition probability matrix are independent and distributed accordingly to Dirichlet distribution, that is, for the *i*th row in the transition probability matrix we choose Dirichlet prior distribution Dirichlet($\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}$) given by

$$f(p_{i1}, \ldots, p_{in}) = \frac{\Gamma(\alpha_{i1} + \ldots + \alpha_{in})}{\Gamma(\alpha_{i1}) \ldots \Gamma(\alpha_{in})} \prod_{j=1}^{n} p_{ij}^{\alpha_{ij} - 1} \tag{4}$$

where $\alpha_{i1}, \ldots, \alpha_{in} > 0$, $p_{ij} \geq 0$, and $\sum_{j=1}^{n} p_{ij} = 1$. As in case of components reliabilities, we use the "ignorance" uniform prior Dirichlet$(1, 1, \ldots, 1)$, which leads to posterior distribution Dirichlet$(r_{i1} + 1, r_{i2} + 1, \ldots, r_{in} + 1)$.

3.2. Model sensitivity to the parameters

The model sensitivity of the system reliability R to component reliability R_i is defined as the partial derivative of R with respect to R_i

$$s_{R_i} = \frac{\partial R}{\partial R_i}. \tag{5}$$

We compute a similar metric with respect to transition probabilities

$$s_{p_{ij}} = \frac{\partial R}{\partial p_{ij}}. \tag{6}$$

These measures of sensitivity represent the impact of changing a specific component reliability or transition probability on the system reliability. Thus, the larger the sensitivity number the greater the impact. The impact of changing a parameter can also be assessed by substituting the means into R for all but one parameter, and then graphing R as a function of only that parameter. Graphing R as a function of any R_i is easy since each R_i can change independently. However, to graph R as a function of p_{ij} we must maintain $\sum_j p_{ij} = 1$ for each i. In other words, in order to consider the effect of varying one transition probability, at least one other transition probability in that row has to be varied.

The upper bound on the model sensitivity to each component reliability (5), obtained when $\partial R/\partial R_i$ is estimated assuming $R_j = 1$ for $1 \leq j \leq n$ and $j \neq i$, is equal to $v_i = (I - Q)_{1i}^{-1}$ where Q is the restriction of the transition probability matrix P to the transient states [16]. It is well known that v_i represents the expected number of visits to state i (i.e., expected number of executions of component i). The true sensitivity of component i will be very close to the upper bound v_i when the component reliabilities are sufficiently close to one.

Based on the above discussion, it follows that if only the model sensitivity is considered, the more often a component is executed the more influence its reliability has on the system reliability. It is important to emphasize that this will not always be the case, since the uncertainty of the parameters also contributes towards the variability of the reliability estimate, and in some cases it may overcome the model sensitivity. Consequently, considering either the model sensitivity or the uncertainty of the parameters in an isolation will not provide a satisfactory explanation of the observed influence of different parameters on the system reliability. The empirical results presented in this paper confirm this theoretical observation.

3.3. Component entropy

We use the approach presented in [12] for uncertainty analysis based on the concept of entropy. Thus, the entropy of component i is defined as the conditional entropy given by

$$H_i = -\sum_{j=1}^{n} p_{ij} \log p_{ij}. \tag{7}$$

In general, the entropy of component i will be higher if it transfers the control to more components (i.e. more states are directly reachable from state i) and the transition probabilities are (close to) equiprobable. Therefore, components with

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higher entropy may be considered critical because they affect larger part of the system.

3.4. Variance of system reliability

One of the main goals of this paper is to explain why (given an operational profile) components reliabilities have more significant impact on the variability of the system reliability estimate than the transition probabilities, an interesting observation made in our earlier work [8], [9], [11]. For this purpose we analyze the equation for the variance of the system reliability derived in [9] using the method of moments. This method consists of expanding $R = f(R_i, p_{ij})$ about $(E[R_1], \ldots, E[R_n], E[p_{11}], \ldots, E[p_{nn}])$, the point at which each of component reliabilities and transition probabilities takes its expected value, by a multivariable Taylor series. The method of moments is an approximate, rather than an exact, method because of the omission of higher order terms in the Taylor series expansion. Thus, the first order Taylor series expansion of the system reliability R is given by

$$R \sim s_0 + \sum_{i=1}^{n} s_{R_i}(R_i – E[R_i]) + \sum_{i=1}^{n}\sum_{j=1}^{n} s_{p_{ij}}(p_{ij} – E[p_{ij}]) \quad (8)$$

where

$$s_0 = f(E[R_1], \ldots, E[R_n], E[p_{11}], \ldots, E[p_{nn}]) \quad (9)$$

$$s_{R_i} = \left.\frac{\partial R}{\partial R_i}\right|_{R_i=E[R_i],\, p_{ij}=E[p_{ij}]} \text{ for } \quad i,j=1,2,\ldots n. \quad (10)$$

$$s_{p_{ij}} = \left.\frac{\partial R}{\partial p_{ij}}\right|_{R_i=E[R_i],\, p_{ij}=E[p_{ij}]} \text{ for } \quad i,j=1,2,\ldots n. \quad (11)$$

Then, the mean and the variance of the system reliability are given by

$$E[R] \sim s_0 \quad (12)$$

$$\begin{aligned}\mathrm{Var}[R] \sim & \sum_{i=1}^{n} s_{R_i}^2 \mathrm{Var}[R_i] + \sum_{k=1}^{n}\sum_{i=1}^{n} s_{p_{ki}}^2 \mathrm{Var}[p_{ki}] \\ & + 2\sum_{k=1}^{n}\sum_{i=1}^{n}\sum_{j=i+1}^{n} s_{p_{ki}} s_{p_{kj}} \mathrm{Cov}(p_{ki}, p_{kj}). \end{aligned} \quad (13)$$

An important observation that helps our analysis is that the coefficients (10) and (11) are equivalent to the model sensitivity parameters given with equations (5) and (6) respectively, estimated at $(E[R_1], \ldots, E[R_n], E[p_{11}], \ldots, E[p_{nn}])$.

Having this in mind, several important theoretical arguments can be made from equation (13). First, the variability of the system reliability estimate clearly depends on both model sensitivity (represented with coefficients s_{R_i} and $s_{p_{ij}}$) and the uncertainty of the parameters' estimates (represented by $\mathrm{Var}[R_i]$, $\mathrm{Var}[p_{ki}]$ and $\mathrm{Cov}(p_{ki}, p_{kj})$).

The second observation provides answer to the question why transition probabilities have significantly smaller influence on the variance of the system reliability. Observe that the first term in the equation (13) expresses the contribution of the component reliabilities, while the second and the third terms express the contribution of the transition probabilities. Note that the term with the covariance $\mathrm{Cov}(p_{ki}, p_{kj})$ appears due to the fact that the transition probabilities are dependent variables (that is, they must sum to 1 for each row of the transition probability matrix). Consequently, whenever a change is made to one transition probability at least one other transition probability has to be changed in an opposite direction. Mathematically, this is expressed in the fact that for the Dirichlet distribution $\mathrm{Cov}(p_{ki}, p_{kj}) < 0$. In other words, even if the model sensitivity and the variance of the transition probabilities are comparable to the model sensitivity and the variance of the component reliabilities, the third term in the equation (13) is always negative and it will decrease the contribution of the transition probabilities to the variance of the system reliability estimate.

It is important to emphasize that the equation for the variance of the system reliability (13), although it accounts for uncertainty of both component reliabilities and transition probabilities, is much simpler and more intuitive than the equation derived in [5] which accounts only for the uncertainty in components reiabilities. This allows us to draw the above general conclusions which apply for any architecture–based software reliability model that uses DTMC to describe the software execution behavior and assumes that components fail independently.

4. Description of case studies and metrics values

We use two case studies: the European Space Agengy (ESA) software which consists of about 10,000 lines of C code and the open source application Indent which consists of about 11,000 lines of C code. Next, we briefly describe how the software architecture and failure behavior were determined and then combined to estimate the system reliability. For detailed description on building the architecture–based software reliability models the reader is referred to [8], [11].

For the ESA study, component traces obtained during testing were used to construct the dynamic software architecture and estimate transition probabilities p_{ij}. Component reliabilities were estimated using fault injection. The faults injected were real faults discovered during testing and operational usage. The DTMC has 3 states, each representing a different subsystem and an additional state representing the *end* of the execution. Using the model the system reliability estimate is 0.7601. The actual reliability is 0.7393.

To determine the dynamic software architecture of Indent, the regression test suite from a later version was ran on an older instrumented version. Running a later version of test cases on older version of the code allows more failures to be observed since tests are often added to the regression test suite after a failure is observed and corrected. The DTMC of Indent has 9 states, each corresponding to one file, and an additional state representing the *end* of the execution. It should be noted that in the case of Indent, the small percentage of failures that led to fixing faults in more that one component were not considered since these failures do not fit into the model assumptions. Using the model the system reliability estimate is 0.8602 and the actual reliability is 0.8378. *It follows that the error of the reliability estimate is less than 3% for both* *studies.*

The motivation for this work is presented in Figures 1 and 2. These figures show the contribution of parameters to the variability of the overall system reliability with respect to the model sensitivity and uncertainty in the parameters estimates for the ESA and Indent case study, respectively. The charts were made based on the results of the Monte Carlo simulations. Specifically, the sampled values of each parameter were ranked, as well as the estimated values of the system reliability. Then, the Pearson's correlation coefficients were estimated for each pair of ranks of the parameter and the corresponding system reliability and averaged over all simulations. Finally, the average values of the Pearson's correlation coefficient for each parameter were squared, normalized by the sum of all squares, and converted to percentages. In Figure 1, two of the six parameters in the ESA study are responsible for contributing 93.2% of the variance; both parameters are component reliabilities. In Figure 2, ten of the 43 parameters in the Indent model are responsible for 99.6% of the variation in the system reliability estimate. All except one of these ten parameters are component reliabilities.

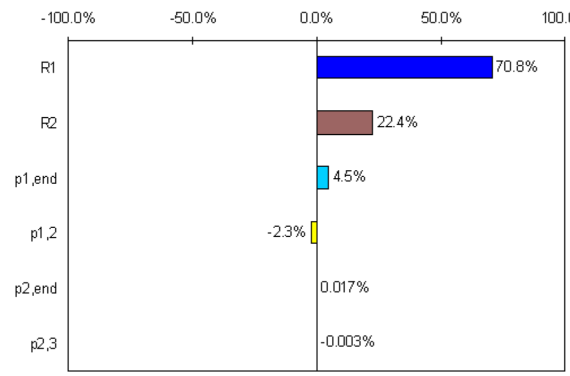

Figure 1. ESA - Contribution of parameters to the variability of the system reliability based on both uncertainty and model sensitivity

In what follows, we address the following research questions related to the observations made from Figures 1 and 2. (1) Why do some component reliabilities matter more than others? (2) Why are component reliabilities more influential than transition probabilities? (3) Do simple sensitivity studies accurately asses the impact of all parameters? First, we present the values for the metrics of each study, and then we provide a summary of the empirical results.

The means, variances, and the coefficients of variability for all parameters of the ESA case study are given in Table 1. It is obvious that these values do not explain well the observations made from Figure 1. Thus, although components 1 and 2 have close values of the moments and coefficients of variability, R_1 has significantly higher impact on the variability of the overall reliability. Furthermore, even though the coefficients of variability of transition probabilities are an order of magnitude higher than those of the component reliabilities, less than 7% of the variability of the reliability is due to transition probabilities.

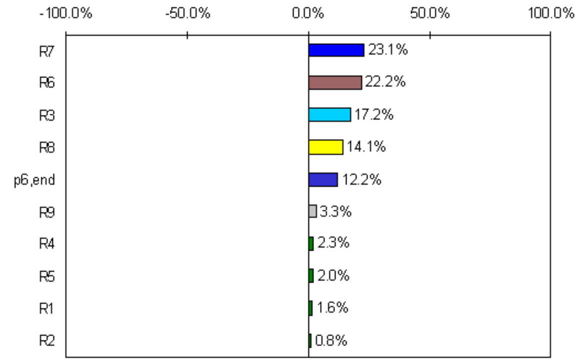

Figure 2. Indent - Contribution of the top ten parameters to the variability of the system reliability based on both uncertainty and model sensitivity

Table 2 shows the mean, variance and coefficient of variability for the 9 components of Indent. Notice that component 2 has the lowest mean reliability and highest coefficient of variability, followed by component 1 with the second lowest mean and the second highest coefficient of variability. The means of the other seven components are larger and the coefficients of variability are significantly smaller.

Parameter	Mean	Variance	Coefficient of variability
R_1	0.8428	0.0064	0.0949
R_2	0.8346	0.0064	0.0959
R_3	1.0000	0.0000	0.0000
$p_{1,2}$	0.5933	0.0224	0.2521
$p_{1,end}$	0.4067	0.0224	0.3678
$p_{2,3}$	0.7704	0.0191	0.1795
$p_{2,end}$	0.2296	0.0191	0.6024

Table 1. ESA - moments of each parameter

Parameter	Mean	Variance	Coefficient of variability
R_1	0.968750	$9.17 \cdot 10^{-04}$	$3.13 \cdot 10^{-02}$
R_2	0.888889	$9.88 \cdot 10^{-03}$	$1.12 \cdot 10^{-01}$
R_3	0.998870	$1.28 \cdot 10^{-07}$	$3.58 \cdot 10^{-04}$
R_4	0.999931	$4.74 \cdot 10^{-09}$	$6.88 \cdot 10^{-05}$
R_5	0.999906	$8.74 \cdot 10^{-09}$	$9.35 \cdot 10^{-05}$
R_6	0.999920	$7.93 \cdot 10^{-10}$	$2.82 \cdot 10^{-05}$
R_7	0.999928	$8.74 \cdot 10^{-10}$	$2.95 \cdot 10^{-05}$
R_8	0.999947	$9.44 \cdot 10^{-10}$	$3.07 \cdot 10^{-05}$
R_9	0.999812	$1.77 \cdot 10^{-08}$	$1.33 \cdot 10^{-04}$

Table 2. Indent - moments of R_i

Moments were estimated for the transition probabilities, but they are not included due to space limitations. The means range from $1.48 \cdot 10^{-03}$ to $8.89 \cdot 10^{-01}$, the variances range from $3.15 \cdot 10^{-09}$ to $3.13 \cdot 10^{-03}$, and the coefficients of variability from $2.44 \cdot 10^{-03}$ to $9.99 \cdot 10^{-01}$. Note that similarly to the ESA case study, the coefficients of variability of transition probabilities are higher than the coefficients of variability of component reliabilities (see Table 2). Nevertheless, only one

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transition probability $p_{6,end}$ is among the top ten contributors to the variability of the overall reliability.

Next, we analyze the ESA and Indent software execution models based on the values of component entropies given in Tables 3 and 4, respectively. For the ESA, the component 1 entropy is greater than component 2 entropy since the probabilities associated with each arc leaving component 1 are closer to equiprobable than those leaving component 2 (see Table 1). The entropy of component 3 is 0 since the control can transfer only to the *end* state. For Indent, components 6 and 7 have the highest component entropies.

Component	Component entropy
1	0.9747
2	0.7773
3	0.0000

Table 3. ESA - components entropies

Component	Component entropy
1	1.0415
2	0.5033
3	1.2169
4	0.7861
5	0.0000
6	1.3975
7	1.4206
8	1.2981
9	1.2679

Table 4. Indent - components entropies

The model sensitivity values and their upper bounds for components reliabilities of the ESA are given in Table 5. As discussed in section 3.2, the upper bounds of s_{R_i} represent how often components are executed. Component 1 has the highest upper bound and the highest true sensitivity value. Figure 3 shows how R varies as a function of each R_i when all remaining parameters are set to their mean values. Notice that the slope of the line for R_1 is steeper leading to higher variation of R (from 0 to 0.9). This explains the fact that the model is more sensitive to R_1 than to R_2.

The values for the model sensitivity due to transition probabilities for ESA are given in Table 6. Figure 4 shows how R changes as each p_{ij} changes, while other parameters are set at their means. Notice that the range of change of R is significantly smaller [0.70,0.85] as transition probabilities vary than when reliabilities vary.

Table 7 shows the model sensitivity values for the component reliabilities in Indent. Components 6, 7, and 8 have the highest true sensitivity and the highest upper bound on sensitivity. Notice that the ranking of components is the same whether true sensitivity or the upper bound on sensitivity is used. Figure 5 shows how R changes as each R_i varies while the remaining parameters are set to their means. Around the means of each R_6, R_7, and R_8 (see Table 1) the slopes are steepest. On the other side, the slopes of R_1 and R_2 are very small. It follows that, for this set of parameters, no matter what the means of R_1 or R_2 are, their effect on the variability of R will not be significant.

We also calculated the sensitivity of the overall reliability

Component reliability	True sensitivity	Upper bound
R_1	0.9019	1.0000
R_2	0.5000	0.5933
R_3	0.3215	0.4571

Table 5. ESA - model sensitivity to R_i

Transition probability	True sensitivity
$p_{1,2}$	0.7034
$p_{1,end}$	0.8428
$p_{2,3}$	0.4173
$p_{2,end}$	0.4173

Table 6. ESA - model sensitivity to p_{ij}

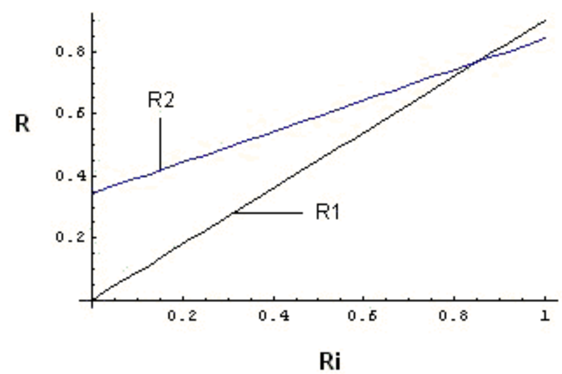

Figure 3. ESA - model sensitivity to R_i

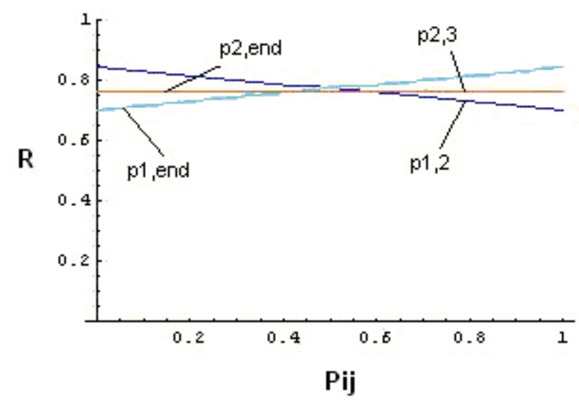

Figure 4. ESA - model sensitivity to p_{ij}

of Indent with respect to each transition probability, but due to space limitations they are not given here. The values range from 0.04 to 553. The highest of all model sensitivity values, including those for reliabilities, is the sensitivity of the transition from component 6 to *end* $s_{p_{6,end}} = 553$, which explains why $p_{6,end}$ is the only transition probability among the top ten contributors to the variability of R.

For transition probabilities, due to space limitations, we only present how system reliability R changes as the transition probability that has the most significant impact from

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Component reliability	True sensitivity	Upper bound
R_1	0.15	0.22
R_2	0.04	0.06
R_3	39.81	59.48
R_4	65.41	97.74
R_5	48.06	71.79
R_6	452.87	676.24
R_7	373.12	557.43
R_8	253.38	378.47
R_9	47.83	71.47

Table 7. Indent - model sensitivity to R_i

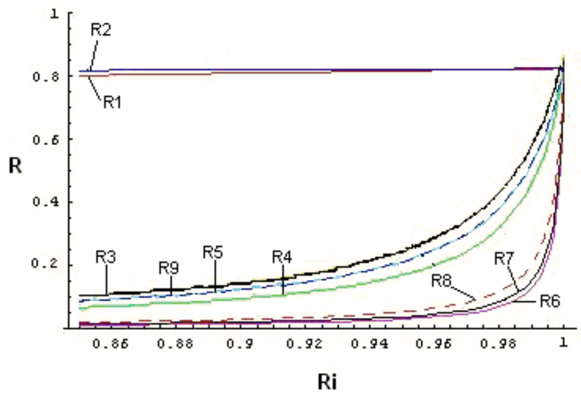

Figure 5. Indent - model sensitivity to R_i

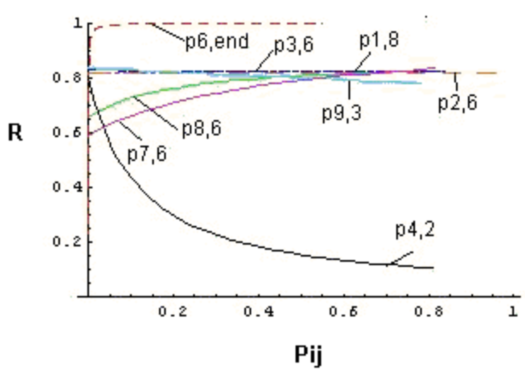

Figure 6. Indent - model sensitivity to p_{ij}

each row varies. To ensure that the probabilities in each row sum to 1, the changes were propagated to one other transition probability in that row and the remaining parameters were assigned their mean values. To no real surprise, as it can be seen from Figure 6, the most influential transition probability is $p_{6,end}$. Notice that the slope is steepest near the mean $p_{6,end} = 0.0015$ and the range of R is from 0 to 0.99.

5. Summary of the empirical results

In this section we summarize the empirical results, specifically addressing the research questions given in Section 4. Figure 1 for ESA case study is fairly simple to explain. Several observations are as follows.

- R_1 has more significant impact on the variability of the system reliability than R_2 (although their moments are close) since it is executed more often (see Table 5). In addition, the entropy of component 1 is higher than that of component 2.

- Component reliabilities contribute more to the variance of the overall reliability than the transition probabilities due to the fact that the range of R as R_i varies is much larger than the range of R when $p_{i,j}$ varies (for any i and any j). With respect to the variance equation (13), 94% comes from the first term (due to component reliabilities) and only 6% comes from the second and third terms (transition probabilities).

- Some transition probabilities such as $p_{2,3}$ have negative contribution to the variance since when their values increase the system reliability R will decrease.

The explanation of the Indent chart given in Figure 2 is more complex and requires more factors to be taken into account.

- R_6, R_7, and R_8 are significant contributors to the variability of the system reliability because these components are executed most often (see Table 7). Also, components 6, 7, and 8 have high entropies. It follows that for these three components the model sensitivity dominates the overall sensitivity.

- The fact that R_3 has more significant impact on the variability of the system reliability estimate than R_8 cannot be explained by the model sensitivity values. Rather, the reason R_3 is so influential is due to the fairly low mean reliability and large coefficient of variability coupled with moderate values of the upper bound on sensitivity (i.e. expected number of executions) and component entropy. Obviously, for component 3 the uncertainty of the parameter overcomes the model sensitivity and results in greater overall sensitivity than component 8.

- On a contrary, although R_1 and R_2 have low means and high coefficients of variability, these components do not have a significant impact on the variability of the system reliability. This can be explained by the fact that the model is not sensitive to components 1 and 2 (see Figure 5), that is, they are executed less often (see Table 7). Thus, in the case of R_1 and R_2 the model sensitivity is more important that the uncertainty of the parameters.

- $p_{6,end}$ is the only transition probability that has a significant impact on the variability of the system reliability due to the fact that it has the highest model sensitivity than any other parameter. However, its contribution to the variability of the system reliability is smaller than some component reliabilities. Following the theoretical argument given in Section 3.4, the estimated contribution to the variance of the system reliability that comes from all transition probabilities is only 19%, which is significantly smaller than the contribution from the component reliabilities.

It should be noted that although the values of the parameters estimated from the empirical studies led to consistent result across multiple case studies about significantly lower contribution of the transition probabilities to the variability of the system reliability, one can come up with hypothetical parameters that will lead to increased importance of transition probabilities (see for example [8]). These values will typically lead to low mean value of the overall reliability.

6. Conclusion

This paper presents an extensive theoretical and empirical study of the variability of the architecture–based software reliability estimates. Unlike other related studies, we consider both the uncertainty in the parameters estimates and the model sensitivity to the parameters to better understand the effects different parameters have on the overall system reliability. In particular, we provide theoretical arguments and support them by the empirical results of two real case studies. Some important observations are as follows. (1) Reliabilities of components that have highest model sensitivities (that is, are executed most often) tend to be among the most influential parameters. (2) However, considering only the model sensitivity often is not sufficient to explain and accurately quantify the effect of parameters. Thus, we observed cases when higher uncertainty in component reliability estimate overcomes the model sensitivity, thus leading to higher impact on the variability of the system reliability than other components with significantly higher model sensitivity, but lower uncertainty of the component reliability estimate. (3) Similarly, considering only the mean and the uncertainty in parameters estimates is not sufficient either. There are cases when low mean and high uncertainty of the component reliability estimate do not lead to high impact on the variability of the system reliability estimate due to extremely low model sensitivity (i.e., low expected number of executions). (4) We explained theoretically the reasons why transition probabilities have smaller impact on the variability of the system reliability than components reliabilities, an interesting empirical phenomenon observed earlier in the related work. In our future work we will conduct similar experiments on additional case studies which will provide basis for generalization of the empirical results.

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