Uncertainty Analysis of Software Reliability Based on Method of Moments [*]

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1 Introduction

Many architecture – based software reliability models have been proposed in the past [2]. Regardless of the accuracy of the mathematical model used to model software reliability, if considerable uncertainty in components failure data exists (as it usually does) then a significant uncertainty in calculated system reliability exists. Therefore, the traditional approach of computing the point estimate of the system reliability by plugging point estimates of component reliabilities into the model is not appropriate. In order to answer the question how parameters uncertainties propagate into overall system reliability, uncertainty analysis is necessary. Several methods for uncertainty analysis of system characteristics from uncertainties in component characteristics are available [4], [5], [6]. In this short paper we use the method of moments to quantify the propagation of uncertainties (i.e. propagation of errors) in software reliability. Method of moments is an approximate approach that allows us to generate the moments of system reliability from the moments of component reliabilities.

2 Method of Moments

Architecture – based software reliability models combine software architecture which describes the manner in which different components interact with components failure behavior specified in terms of their reliabilities or failure rates. The method of moments can be applied to any architecture – based software reliability model that has a close form solution for the system reliability. In this paper we will use the model first presented in [1] to obtain the relationship between system reliability R and the component reliabilities $R_1, R_2, \ldots, R_n$ given by the function $R = f(R_1, R_2, \ldots, R_n)$. The actual relationship between system reliability and components reliabilities depends on the specific software architecture. If we treat each component reliability on the right – hand side of this expression as a random variable, then th system reliability is also a random variable. Let $E[R_i]$ be the mean value of the ith component reliability and let $\mu_k[R_i]$ denote its kth central moment (or moment about the mean). The method of

moments allows us to obtain the estimates of the expected value $E[R]$ and kth central moments $\mu_k[R]$ for system reliability based on (1) knowledge of the system structure $R = f(R_1, R_2, \ldots, R_n)$ and (2) data on the components reliabilities from which estimates of $E[R_i]$ and $\mu_k[R_i]$ for $i = 1, 2, \ldots, n$ can be obtained.

System reliability moments are generated by expanding the system function $R = f(R_1, R_2, \ldots, R_n)$ in a multivariable Taylor series expansion about the statistically expected values of each of the component reliabilities $E[R_i]$. We have used *Mathematica* to derive the system reliability expression $R = f(R_1, R_2, \ldots, R_n)$ and its partial derivates for the Taylor series expansion.

The method of moments is an approximate, rather than an exact, method, because of the omission of higher order terms in the Taylor series expansion. Thus, the first order Taylor series expansion is given by

$$R \sim a_0 + \sum_{i=1}^{n} a_i(R_i - E[R_i]) \qquad (1)$$

where

$$a_0 = f(E[R_1], E[R_2], \ldots, E[R_n])$$
$$a_i = \left. \frac{\partial R}{\partial R_i} \right|_{R_i = E[R_i]} \text{ for } i = 1, 2, \ldots n$$

Then, the mean and the variance of system reliability are given by $E[R] \sim a_0$ and $Var[R] = \mu_2[R] \sim \sum_{i=1}^{n} a_i^2 Var[R_i]$.

The accuracy of the $E[R]$ and $Var[R]$ can be improved by including higher order terms in the Taylor series expansion. We have also derived the second order Taylor series expansion and the expressions for the mean and the variance of system reliability, but they are omitted here due to space limitation. Note that generating the mean and the variance of system reliability from the second order Taylor series expansion requires the knowledge of the first four central moments of component reliabilities. Even more, we can generate the first four central moments of the system reliability using the first eight central moments of component reliabilities. Then, the estimates of the first four moments may be used to select an empirical distribution from which the percentiles of the system reliability distribution may be obtained.

Next, we illustrate the method of momemnts on the case

		First order Taylor series	Second order Taylor series
	Mean	0.7601	0.7601
Version A	Variance	0.0068	0.0068
	C_R	0.1085	0.1085
	Mean	0.8782	0.8782
Version B	Variance	0.0035	0.0035
	C_R	0.0671	0.0671

Table 1. Mean and variance of the system reliability for the case study presented in [3]

		First order Taylor series	Second order Taylor series
	Mean	0.6261	0.6314
Version C	Variance	0.0106	0.0101
	C_R	0.1640	0.1589

Table 2. Mean and variance of the system reliability for the hypothetical example presented in [3]

study presented in [3]. The software application consisting of 10,000 lines of C code was modeled by four state discrete time Markov chain and the expression for system reliability was derived as

$$R = (1 - p_{12})R_1 + p_{12}(1 - p_{23})R_1R_2 + p_{12}p_{23}R_1R_2R_3.$$

In the empirical study, two faulty versions of this program were obtained using fault injection: version A with two faulty components and version B with only one faulty component.

Table 1 compares the values obtained for the mean, variance and coefficient of variation C_R (a relative measure of the spread of the distribution) of the system reliability for versions A and B using first and second order Taylor series expansion. As expected, version B has higher mean reliability then version A. In addition, the variance is smaller and the distribution of the system reliability is less spread. Further, for this example the second order approximation does not improve the accuracy.

In general, higher order Taylor series expansion will increase accuracy, as it can be seen form Table 2 which presents the results obtained for the hypothetical example from the same paper [3] (referred here as Version C).

Although the accuracy may be further increased, the derivation of the third or higher order approximations would constitute a formidable task and require higher number of central moments for component reliabilities. Even if the expressions for the third (or higher) order approximation are derived, it might happen that the sampling error due to lim-

ited number of observations available for estimation of the central moments of the component reliabilities will exceed the error introduced by the omission of higher order terms.

3 Concluding remarks

The general goal of this paper is to point out the need for conducting uncertainty analysis in software reliability. In particular, we have presented the method of moments, one of the several methods collectively referred to as the propagation of uncertainty. The method of moments has several advantages. First, it requires only the knowledge of the moments of components reliabilities, that is, no distribution function must be specified. Second, generation of random numbers is not required, therefore there is no sampling error. Finally, it could be applied to dependent as well as independent parameters, although the expressions for dependent variables would be more difficult to derive due to their complexity.

However, the method is approximate and a finite error is associated with the use of only up to first (second) order terms in the Taylor series expansion. Further, the accuracy of this method is not readily quantifiable. Therefore, if a precise accuracy calculations for system reliability are required to support the uncertainty analysis, the method of moments might not be a good choice. Our current research is focused on implementing the Monte Carlo simulations for the uncertainty analysis and comparing the results with the results from the method of moments presented in this paper.

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