GORENSTEIN RINGS AND CONNECTED SUMS

Abstract. In topology, amalgamating two manifolds M and N near a chosen point on each creates another, usually distinct, manifold $M\#N$, called a "*connected sum*". This concept plays a significant role in the classification of closed surfaces.

In algebra the motivation for studying connected sums come from the theory of manifolds: the connected sum $R\#_kS$ of a Gorenstein Artin local ring R and S over their common residue field k mimic the cohomology algebra $H^*(M\#N)$ of connected sums of compact smooth manifolds in terms of the cohomology algebras $H^*(M)$ and $H^*(N)$. In fact, this relation is the main inspiration of defining the connected sums of rings.

Gorenstein rings, due to their various kinds of symmetries and duality properties, form an important class of rings. The origins of Gorenstein rings go back to the classical study of plane curves, and such rings are now part of the basic landscape of mathematics. A search for the word "Gorenstein ring" in MathSciNet will reveal hundreds of entries.

In 2012 Ananthnarayan, Avramov and Moore introduced a new construction of Gorenstein rings. They defined a *connected sum* of two Gorenstein local rings as an appropriate quotient of their fiber product. Although the fiber product is rarely Gorenstein, a connected sum of two Gorenstein local rings is always a Gorenstein ring.

In this talk we discuss the history and the ubiquity of Gorenstein rings and give some examples. Following this discussion, we will explore the properties of connected sums $R\#_kS$ of Gorenstein Artin local rings R and S over their common residue field k.