

Fuzzy connectedness segmentations with different non-symmetric affinities

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Based mainly on a joint work with Gabor T. Herman and T. Yung Kong

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Preamble: What this talk is about

- The subject of this talk is theoretical, no experimental part
- Concerns an image segmentation theory that unifies two **Fuzzy Connectedness, FC**, tracks:
 - **(I)RFC**, on RFC and IRFC segmentations, favored by MIPG
 - **MOFS**, favored by Herman, Kong, and others
- Optimized algorithm for finding **MOFS and IRFC** objects which is **the most efficient** among existing algorithms finding MOFS or IRFC (that have no tie-zones ambiguity)
- All presented results concern “hard” segmentations, as opposed to fuzzy segmentations (Though, they could be easily reformulated to fuzzy sets framework)

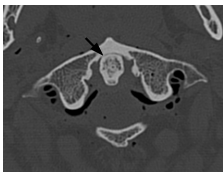
Outline

- 1 Image segmentation: example, definitions
- 2 Basics of FC theory
- 3 RFC, IRFC, and MOFS objects defined via algorithms
- 4 Getting IRFC objects from MOFS objects
- 5 Characterizations of RFC, IRFC, and MOFS objects
- 6 Robustness of MOFS (and (I)RFC) objects on seeds choice
- 7 Efficient algorithm for finding MOFS (and IRFC) objects
- 8 Some illuminating examples
- 9 Summary

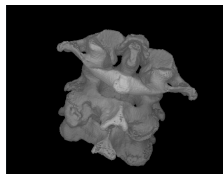
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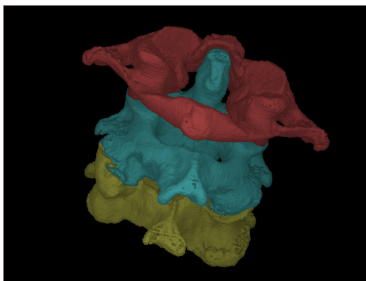
Image segmentation example: CT, cervical spine



A slice of an original 3D image



Surface rendition of segmented three vertebrae, together



Color surface rendition of the segmented three vertebra

Image segmentation — formal setting

- An *image* is a map f from a set V (of spels) into \mathbb{R}^k
The value $f(c)$ represents **image intensity at c** , a k -dimensional vector each component of which indicates a measure of some aspect of the signal, like color.
- **Segmentation problem**: Given an image $f: V \rightarrow \mathbb{R}^k$, find a “**desired**” family $\{O_1, \dots, O_M\}$ of subsets of V .
- We will assume the objects are indicated by disjoint sets S_i of **seeds**, imposing that $S_i \subset O_i \subset V \setminus \bigcup_{j \neq i} S_j$.
- We impose neither that O_i 's are disjoint nor that $V = \bigcup_i O_i$.

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Affinities: concise information coding of the image

In FC framework, all information on the image and (beside the seed sets) the objects is coded via **affinity function(s)**:

maps $\psi: V \times V \rightarrow [0, 1]$ such that $\psi(v, v) = 1$ for all $v \in V$.

The larger $\psi(c, d)$, the stronger c is (locally ψ -) connected to d .

No adjacency: for non-adjacent c and d we put $\psi(c, d) = 0$.

(Though adjacency/edge reappears in efficient algorithms.)

In **MOFS track**, each i th object has its own affinity ψ_i .

In **(I)RFC track**, there is single affinity ψ for all objects.

Even for **(I)RFC** results, we do not assume ψ is symmetric.
(Symmetric means $\psi(c, d) = \psi(d, c)$ for all $c, d \in V$.)

Path strength and connectivity via $W \subset V$

Given affinity $\psi: V \times V \rightarrow [0, 1]$ and $A, B, W \subseteq V$,

- a *W-path* from A to B : any sequence $p = \langle w_0, \dots, w_\ell \rangle$ of points in W such that $w_0 \in A$ and $w_\ell \in B$;
- the *ψ -strength* of $p = \langle v_0, \dots, v_\ell \rangle$ is: $\psi(p) = 1$ if $\ell = 0$ and $\psi(p) = \min_{1 \leq j \leq \ell} \psi(v_{j-1}, v_j)$ if $\ell > 0$;

$$\psi^W(A, B) = \max \{ \psi(p) \mid p \text{ is a } (W \cup A \cup B)\text{-path from } A \text{ to } B \}.$$

- Seed sets S_1, \dots, S_M are *consistent* with the affinities ψ_1, \dots, ψ_M if $\psi_i^V(S_i, S_j) < 1$ for all $i \neq j$.

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The rationale behind the following three algorithms

- to **emphasize great similarities among (I)RFC and MOFS segmentations**
- to use their outputs as unambiguous **definitions of RFC, IRFC and MOFS objects** (there were some small ambiguities in definitions of IRFC, especially in IFT setting)
- to facilitate parallel development of the mathematics of (I)RFC and MOFS segmentations
- these three algorithms are **not optimized for computation** of either of the FC segmentations

RFC objects

Algorithm 1: RFC Segmentation into M Objects O_i^{RFC}

Data: Seed sets $S_1, \dots, S_M \subset V$; an affinity ψ on V

Result: The RFC segmentation $\langle O_1^{\text{RFC}}, \dots, O_M^{\text{RFC}} \rangle$ of V

```

1 for  $i \leftarrow 1$  to  $M$  do  $T_i \leftarrow S_i$ 
2 sort  $A = \psi[V \times V] \setminus \{0\}$  into  $1 = \alpha_1 > \dots > \alpha_{|A|}$ 
3 for  $n \leftarrow 1$  to  $|A|$  do /* the main loop */
4   for  $i \leftarrow 1$  to  $M$  do
      $newT_i \leftarrow T_i \cup \{v \in V \setminus \bigcup_j T_j \mid \psi^V(T_i, v) \geq \alpha_n\}$ 
5   for  $i \leftarrow 1$  to  $M$  do  $T_i \leftarrow newT_i$ 
6 for  $i \leftarrow 1$  to  $M$  do  $O_i^{\text{RFC}} \leftarrow T_i \setminus \bigcup_{j \neq i} T_j$ 

```

IRFC objects

Algorithm 2: IRFC Segmentation into M Objects O_i^{IRFC}

Data: Seed sets $S_1, \dots, S_M \subset V$; an affinity ψ on V

Result: The IRFC segmentation $\langle O_1^{\text{IRFC}}, \dots, O_M^{\text{IRFC}} \rangle$ of V

```

1 for  $i \leftarrow 1$  to  $M$  do  $T_i \leftarrow S_i$ 
2 sort  $A = \psi[V \times V] \setminus \{0\}$  into  $1 = \alpha_1 > \dots > \alpha_{|A|}$ 
3 for  $n \leftarrow 1$  to  $|A|$  do /* the main loop */
4   for  $i \leftarrow 1$  to  $M$  do
      $newT_i \leftarrow T_i \cup \{v \in V \setminus \bigcup_j T_j \mid \psi^{V \setminus \bigcup_j T_j}(T_i, v) \geq \alpha_n\}$ 
5   for  $i \leftarrow 1$  to  $M$  do  $T_i \leftarrow newT_i$ 
6 for  $i \leftarrow 1$  to  $M$  do  $O_i^{\text{IRFC}} \leftarrow T_i \setminus \bigcup_{j \neq i} T_j$ 

```

MOFS objects

Algorithm 3: MOFS Segmentation into M Objects O_i^{MOFS}

Data: Seed sets $S_1, \dots, S_M \subset V$; M affinities ψ_1, \dots, ψ_M on V

Result: The MOFS segmentation $\langle O_1^{\text{MOFS}}, \dots, O_M^{\text{MOFS}} \rangle$ of V

```

1 for  $i \leftarrow 1$  to  $M$  do  $T_i \leftarrow S_i$ 
2 sort  $A = \bigcup_j \psi_j[V \times V] \setminus \{0\}$  into  $1 = \alpha_1 > \dots > \alpha_{|A|}$ 
3 for  $n \leftarrow 1$  to  $|A|$  do                               /* the main loop */
4   for  $i \leftarrow 1$  to  $M$  do
      $newT_i \leftarrow T_i \cup \{v \in V \setminus \bigcup_j T_j \mid \psi_i^{\bigvee \bigcup_j T_j}(T_i, v) \geq \alpha_n\}$ 
5   for  $i \leftarrow 1$  to  $M$  do  $T_i \leftarrow newT_i$ 
6 for  $i \leftarrow 1$  to  $M$  do  $O_i^{\text{MOFS}} \leftarrow T_i$ 

```

Corollary and Correctness Theorem

Corollary (IRFC from MOFS)

If $\psi_1 = \dots = \psi_M = \psi$, then $O_i^{\text{IRFC}} = O_i^{\text{MOFS}} \setminus \bigcup_{j \neq i} O_j^{\text{MOFS}}$ for all i .

The equation is true even when ψ is not symmetric.

Any algorithm finding MOFS objects finds also IRFC objects!

(This is important, as our optimized MOFS finding algorithm is more efficient than currently existing IRFC finding algorithms.)

Theorem (Correctness of the algorithms)

If the seed sets S_1, \dots, S_M are consistent with the affinities ψ_1, \dots, ψ_M , then **Algorithm 3** returns the same **MOFS objects** that were found by older algorithms.

If, in addition, $\psi_1 = \dots = \psi_M = \psi$ and ψ is symmetric, then **Algorithms 1 and 2** return the same **RFC and IRFC objects** that were found by older algorithms.

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IRFC from MOFS, Round I

- If $\psi_1 = \dots = \psi_M = \psi$ and ψ is symmetric, then

$$O_i^{\text{IRFC}} = O_i^{\text{MOFS}} \setminus \bigcup_{j \neq i} O_j^{\text{MOFS}}$$
- If $\psi_1 = \dots = \psi_M = \psi$ and ψ is non-symmetric, then we can still define $O_i^{\text{IRFC}} \stackrel{\text{def}}{=} O_i^{\text{MOFS}} \setminus \bigcup_{j \neq i} O_j^{\text{MOFS}}$
 These new IRFC objects will still have good properties.
- If ψ_i 's are distinct, even symmetric, then objects
 $O_i = O_i^{\text{MOFS}} \setminus \bigcup_{j \neq i} O_j^{\text{MOFS}}$ will not have good properties.
 (Can be disconnected from the seeds – example latter.)
- With ψ_i 's distinct,
can we ensure that the objects O_i^{MOFS} are disjoint?
 Then such objects would have good properties and could be used as new IRFC objects.

IRFC from MOFS, Round II

Theorem

If S_1, \dots, S_M are consistent with the affinities ψ_1, \dots, ψ_M , then objects O_i^{MOFS} are disjoint as long as

(D) for any $c \neq d$, $u \neq v$, and $i \neq j$,

$\psi_i(c, d) = \psi_j(u, v)$ can happen only when $\psi_i(c, d) = 0$.

In such case, O_i^{MOFS} can be treated as IRFC objects.

Property (D) can be insured by perturbing ψ_i 's as follows:

if $M = 4$, $\psi_i(u, v) < 1$ for $u \neq v$, and values $\psi_i(u, v)$ are floating point binary values, then for any $u \neq v$ with $\psi_i(u, v) > 0$ set the two least significant bits of $\psi_i(u, v)$ to 00, 01, 10, or 11 for $i = 1, 2, 3, 4$, respectively.

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Mathematical Characterizations of RFC and IRFC

Theorem (RFC, new only for non-symmetric ψ)

Assuming S_1, \dots, S_M are consistent with ψ , the RFC object O_i^{RFC} given by S_1, \dots, S_M and ψ satisfies

$$O_i^{\text{RFC}} = \{v \in V \mid \max_{j \neq i} \psi^V(S_j, v) < \psi^V(S_i, v)\}, \quad (1)$$

$$O_i^{\text{RFC}} = \{v \in V \mid \max_{j \neq i} \psi^V(S_j, v) < \psi^{O_i^{\text{RFC}}}(S_i, v)\}. \quad (2)$$

Theorem (IRFC, new only uniqueness & non-symmetric case)

Assuming S_1, \dots, S_M are consistent with ψ , the IRFC object O_i^{IRFC} given by S_1, \dots, S_M and ψ is the unique set O such that

$$O = \{v \in V \mid \max_{j \neq i} \psi^{V \setminus O}(S_j, v) < \psi^V(S_i, v)\}. \quad (3)$$

Moreover, O_i^{IRFC} is also the unique set O that

$$O = \{v \in V \mid \max_{j \neq i} \psi^{V \setminus O}(S_j, v) < \psi^O(S_i, v)\}. \quad (4)$$

Corollaries on RFC and IRFC

Since $\psi^V(S_j, v) \geq \psi^{V \setminus O}(S_j, v)$, (1) and (3) immediately imply

Corollary (containment, new only for non-symmetric ψ)

$O_i^{\text{RFC}} \subseteq O_i^{\text{IRFC}}$ for all i .

Also (2) and (4) immediately imply

Corollary (connectedness, new only for non-symmetric ψ)

Any v belonging to $O_i \in \{O_i^{\text{RFC}}, O_i^{\text{IRFC}}\}$ is connected to the object's seed set via an internal path of strength $\psi^{O_i}(S_i, v) > 0$.

Mathematical Characterizations of IRFC and MOFS

Theorem (IRFC, repetition)

The IRFC object O_i^{IRFC} given by S_1, \dots, S_M and ψ is the unique set O such that

$$O = \{v \in V \mid \max_{j \neq i} \psi^{V \setminus O}(S_j, v) < \psi^V(S_i, v)\} \text{ and} \quad (5)$$

$$O = \{v \in V \mid \max_{j \neq i} \psi^{V \setminus O}(S_j, v) < \psi^O(S_i, v)\}. \quad (6)$$

Theorem (MOFS, new only part (8))

Assuming S_i 's are consistent with ψ_i 's, the sequence $\langle O_1^{\text{MOFS}}, \dots, O_M^{\text{MOFS}} \rangle$ given by S_i 's and ψ_i 's is the unique sequence of sets $\langle O_1, \dots, O_M \rangle$ such that

$$O_i = \{v \in V \mid \max_{j \neq i} \psi_j^{O_j}(S_j, v) \leq \psi_i^{O_i}(S_i, v) \neq 0\} \text{ for all } i. \quad (7)$$

Moreover, if $\psi_1 = \dots = \psi_M = \psi$, then

$$O_i^{\text{IRFC}} = O_i^{\text{MOFS}} \setminus \bigcup_{j \neq i} O_j^{\text{MOFS}} \text{ for all } i. \quad (8)$$

Corollaries on IRFC and MOFS

(8) immediately implies

Corollary (containment, new only for non-symmetric ψ)

If $\psi_1 = \dots = \psi_M = \psi$, then $O_i^{\text{IRFC}} \subseteq O_i^{\text{MOFS}}$ for all i .

Also (7) immediately implies

Corollary (connectedness, not new at all)

Any v belonging to O_i^{MOFS} is connected to the object's seed set via an internal path of strength $\psi^{O_i^{\text{MOFS}}}(S_i, v) > 0$.

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Robustness of MOFS on increasing sets of seeds

Let $\Psi = \langle \psi_1, \dots, \psi_M \rangle$ be a sequence of affinities;

$\mathcal{S} = \langle \mathcal{S}_1, \dots, \mathcal{S}_M \rangle$ — a sequence of seeds consistent with Ψ ;

$O_i^{\text{MOFS}}(\Psi, \mathcal{S})$ for the i th MOFS object given by Ψ and \mathcal{S} ;

$\text{Core}_i^{\Psi, \mathcal{S}} \subseteq O_i^{\text{MOFS}}(\Psi, \mathcal{S})$ — a “big” set to be described latter.

Theorem (Robustness of MOFS on increasing sets of seeds)

Let $\mathcal{R} = \langle R_1, \dots, R_M \rangle$ be such that $\mathcal{S}_i \subseteq R_i \subseteq \text{Core}_i^{\Psi, \mathcal{S}}$ for $1 \leq i \leq M$. Then the sequence \mathcal{R} is consistent with the affinities and $O_i^{\text{MOFS}}(\Psi, \mathcal{R}) = O_i^{\text{MOFS}}(\Psi, \mathcal{S})$ for $1 \leq i \leq M$.

Robustness of MOFS objects on seeds choice

$\mathcal{S} = \langle S_1, \dots, S_M \rangle$ — a sequence of seeds consistent with Ψ ;

$O_i^{\text{MOFS}}(\Psi, \mathcal{S})$ for the i th MOFS object given by Ψ and \mathcal{S} ;

Theorem (Robustness on increasing sets of seeds, repetition)

Let $\mathcal{R} = \langle R_1, \dots, R_M \rangle$ be such that $S_i \subseteq R_i \subseteq \text{Core}_i^{\Psi, \mathcal{S}}$ for $1 \leq i \leq M$. Then the sequence \mathcal{R} is consistent with the affinities and $O_i^{\text{MOFS}}(\Psi, \mathcal{R}) = O_i^{\text{MOFS}}(\Psi, \mathcal{S})$ for $1 \leq i \leq M$.

Corollary (Robustness of MOFS objects on seeds choice)

Let $\mathcal{S}^* = \langle S_1^*, \dots, S_M^* \rangle$ be any sequence of nonempty sets such that $S_i^* \subseteq \text{Core}_i^{\Psi, \mathcal{S}}$ and $S_i \subseteq \text{Core}_i^{\Psi, \mathcal{S}^*}$ for $1 \leq i \leq M$. Then we have that $O_i^{\text{MOFS}}(\Psi, \mathcal{S}^*) = O_i^{\text{MOFS}}(\Psi, \mathcal{S})$ for $1 \leq i \leq M$.

How big $\text{Core}_i^{\Psi, \mathcal{S}}$ is?

Theorem (On the size of $\text{Core}_i^{\Psi, \mathcal{S}}$)

The set $Q_i^{\Psi, \mathcal{S}}$ of all $v \in O_i^{\text{MOFS}}(\Psi, \mathcal{S}) \setminus \bigcup_{j \neq i} O_j^{\text{MOFS}}(\Psi, \mathcal{S})$ such that $\psi_i^{O_i^{\text{MOFS}}(\Psi, \mathcal{S})}(\mathcal{S}_i, v) \geq \psi_i^V(v, \bigcup_{j \neq i} O_j^{\text{MOFS}}(\Psi, \mathcal{S}))$ is contained in $\text{Core}_i^{\Psi, \mathcal{S}}$.

For symmetric ψ_i , $\text{Core}_i^{\Psi, \mathcal{S}} = Q_i^{\Psi, \mathcal{S}}$.

If all the affinities ψ_i are equal to the same symmetric affinity ψ , then $\text{Core}_i^{\Psi, \mathcal{S}} = O_i^{\text{IRFC}}(\psi, \mathcal{S})$.

This fact and previous theorem **imply** immediately

robustness results for IRFC segmentation!

What is missing in the robustness results

- We do not have any experimental studies on the size of $O_i^{\text{MOFS}}(\Psi, \mathcal{S})$.
- This would be of interest even only in the case of a single non-symmetric affinity.

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Efficient algorithm: Initialization

Algorithm 4: Finds O_i^{MOFS} and $\psi_i^{O_i^{\text{MOFS}}}(S_i, v)$ for each $v \in O_i^{\text{MOFS}}$

Data: seeds $\mathcal{S} = \langle S_1, \dots, S_M \rangle$ and affinities $\Psi = \langle \psi_1, \dots, \psi_M \rangle$

Result: an array $\sigma[\cdot]$, Boolean arrays $\chi^1[\cdot], \dots, \chi^M[\cdot]$ such that,

$\sigma[v]$ = the approximation of $\psi_i^{O_i^{\text{MOFS}}}(S_i, v)$

$\chi^i[v]$ = **true** if v can belong to O_i^{MOFS} ; **false** otherwise

foreach $v \in V$ **do** /* initialization loop 1 */

$\sigma[v] \leftarrow 0$

for $i \leftarrow 1$ **to** M **do** $\chi^i[v] \leftarrow$ **false**

for $i \leftarrow 1$ **to** M **do** /* initialization loop 2 */

foreach $s \in S_i$ **do**

$\sigma[s] \leftarrow 1$

$\chi^i[s] \leftarrow$ **true**

$H \leftarrow V$

Efficient algorithm: Main Loop

```

1 while  $H \neq \emptyset$  do                                     /* the main loop */
2   remove an element  $w$  of  $\arg \max_{u \in H} \sigma[u]$  from  $H$ 
3   foreach  $x$  such that  $(w, x)$  is a  $\Psi$ -edge do
4     foreach  $i \in \{1, \dots, M\}$  such that  $\chi^i[w] = \text{true}$  do
5        $\sigma' \leftarrow \min(\sigma[w], \psi_i(w, x))$ 
6       if  $\sigma' > \sigma[x]$  then
7          $\sigma[x] \leftarrow \sigma'$ 
8         for  $j \leftarrow 1$  to  $M$  do  $\chi^j[x] \leftarrow \text{false}$ 
9          $\chi^i[x] \leftarrow \text{true}$ 
10      else if  $\sigma' = \sigma[x]$  and  $\sigma' > 0$  and  $\chi^i[x] = \text{false}$  then
11         $\chi^i[x] \leftarrow \text{true}$ 
12        if  $x \notin H$  then  $H \leftarrow H \cup \{x\}$ 

```

Correctness and Efficiency of Algorithm 4

Theorem

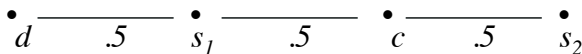
- Algorithm 4 indeed finds O_i^{MOFS} and $\psi_i^{O_i^{\text{MOFS}}}(S_i, v)$ for each i and $v \in O_i^{\text{MOFS}}$.
- In general, its running time $O(|V| \log |V|)$.
- Moreover, if all values of ψ_i 's are multiples of $1/N$ for an integer N that is $O(|V|)$, then we can use an array of doubly-linked lists instead of a heap to represent H , in which case **the running time of Algorithm 4 will be $O(|V|)$** for any given value of M .

Notice, that this last estimate is better than for the best IRFC algorithm existing up to this point, as **the previously best algorithm required $O(M|V|)$ (or $O(M|V| \log |V|)$) operations** for comparable task.

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$O_i^{\text{RFC}} \subsetneq O_i^{\text{IRFC}} \subsetneq O_i^{\text{MOFS}}$, single symmetric ψ



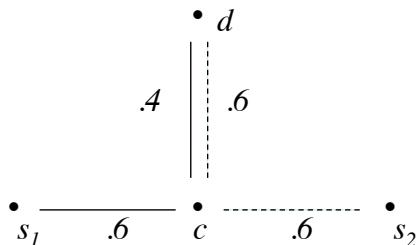
Example

With V and the symmetric affinity ψ as in Figure and $M = 2$,

$$O_1^{\text{RFC}} = \{s_1\} \subsetneq O_1^{\text{IRFC}} = \{s_1, d\} \subsetneq O_1^{\text{MOFS}} = \{s_1, c, d\} \text{ and}$$

$$O_2^{\text{RFC}} = O_2^{\text{IRFC}} = \{s_2\} \subsetneq O_2^{\text{MOFS}} = \{s_2, c\}.$$

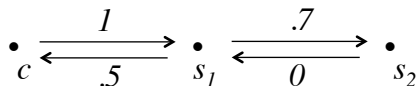
$O_i^{\text{IRFC}} = O_i^{\text{MOFS}} \setminus \bigcup_{j \neq i} O_j^{\text{MOFS}}$ makes no sense for different symmetric affinities ψ_j



Example

With V and the symmetric affinities ψ_i as in Figure, $M = 2$, we get $O_1^{\text{MOFS}} = \{s_1, c\}$ and $O_2^{\text{MOFS}} = \{s_2, c, d\}$. So, object $O_2^{\text{IRFC}} = O_2^{\text{MOFS}} \setminus O_1^{\text{MOFS}} = \{s_2, d\}$ is “disconnected.”

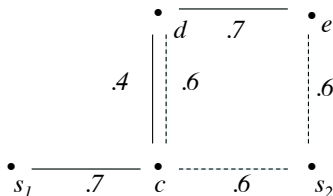
Core_{*i*}^{ψ, S} ≠ Q_{*i*}^{ψ, S} for single non-symmetric affinity



Example

With V and a single non-symmetric affinity as in Figure, $M = 2$. Then $\text{Core}_1^{\psi, S} = \{s_1, c\} = O_1^{\text{MOFS}}$. But $c \notin Q_1^{\psi, S}$ because the ψ -strength of a ψ -strongest V -path from c to S_2 is 0.7, which exceeds the ψ -strength of a ψ -strongest V -path from S_1 to c .

$Q_i^{\Psi, \mathcal{S}} \neq O_i^{\text{MOFS}}(\Psi, \mathcal{S}) \setminus \bigcup_{j \neq i} O_j^{\text{MOFS}}(\Psi, \mathcal{S})$ for different symmetric affinities ψ_j



Example

With V and the symmetric affinities ψ_j as in Figure, $M = 2$.

Then $O_1^{\text{MOFS}}(\Psi, \mathcal{S}) = \{s_1, c, d\}$ and $O_2^{\text{MOFS}}(\Psi, \mathcal{S}) = \{s_2, e\}$, so $O_1^{\text{MOFS}}(\Psi, \mathcal{S}) \setminus \bigcup_{j \neq 1} O_j^{\text{MOFS}}(\Psi, \mathcal{S}) = O_1^{\text{MOFS}}(\Psi, \mathcal{S}) = \{s_1, c, d\}$.

But the set $\text{Core}_1^{\Psi, \mathcal{S}} = Q_1^{\Psi, \mathcal{S}} = \{s_1, c\}$ is smaller.

For $R_1 = O_1^{\text{MOFS}}(\Psi, \mathcal{S}) \not\subseteq \text{Core}_1^{\Psi, \mathcal{S}}$ and $R_2 = S_2$ no robustness: $O_1^{\text{MOFS}}(\Psi, \langle R_1, R_2 \rangle) = \{s_1, c, d, e\} \neq O_1^{\text{MOFS}}(\Psi, \mathcal{S})$.

Outline

- 1 Image segmentation: example, definitions
- 2 Basics of FC theory
- 3 RFC, IRFC, and MOFS objects defined via algorithms
- 4 Getting IRFC objects from MOFS objects
- 5 Characterizations of RFC, IRFC, and MOFS objects
- 6 Robustness of MOFS (and (I)RFC) objects on seeds choice
- 7 Efficient algorithm for finding MOFS (and IRFC) objects
- 8 Some illuminating examples
- 9 Summary

Summary

- We described a general theory of FC segmentations that elegantly encompasses RFC, IRFC, and MOFC tracks.
- We showed how to use MOFS segmentations to immediately find IRFC segmentations in case where we have single symmetric affinity.
- We proposed extension of IRFC segmentation theory to the cases when we work with
 - a single non-symmetric affinity;
 - different affinities with essentially disjoint ranges $\psi_i[V \times V]$.
- We presented an algorithm for finding MOSF (so, IRFC) segmentations more efficient than older IRFC algorithms.
- We extended seeds placement robustness results for IRFC general case of MOFS segmentations.

Thank you for your attention!