

TEST # 2 Review

To be solved in class.

Test will start with: "Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)"

Ex. 1. Find a basis for a vector space V generated by the following vectors:

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 5 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 4 \end{bmatrix}, \text{ and } \vec{v}_3 = \begin{bmatrix} 0 \\ 4 \\ 2 \\ -2 \end{bmatrix}.$$

Ex. 2. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 1 & 2 & 2 \\ 5 & 4 & -2 \end{bmatrix}$. (a) For what value of number c vector $\vec{w} = \begin{bmatrix} 2 \\ -4 \\ 0 \\ c \end{bmatrix}$ belongs

to the column space of A ? (b) Find the most general solution of $A\vec{x} = \vec{w}$ for this value of c . (c) Without further calculation, identify a basis for a row space of A (i.e., for $C(A^T)$).

Ex. 3. Let A be an 7×11 matrix with the rank $r = 4$. What is the dimension of the following four spaces. (a) Column space $C(A)$ of A . (b) Row space $C(A^T)$ of A . (c) Null space $N(A)$ of A . (d) Left-null space $N(A^T)$ of A .

Ex. 4. Let A be an $m \times n$ matrix of rank r . Describe precisely the possible number of solutions of a system $A\vec{x} = \vec{b}$ under the following assumptions:

- (a) $r < m$ and $r < n$
- (b) $r = m$ and $r < n$
- (c) $r < m$ and $r = n$
- (d) $r = m = n$

Ex. 5. Finish the following sentence. Give a short explanation for your answer.

An $n \times n$ matrix A is invertible if, and only if, the dimension of its column space $C(A)$ is ...

In the test, there will be also a bonus exercise.