Math 441 Exam 2

1. Answer the following simple questions, and provide a brief explanation.

a) If the vectors in the set $S = \{\bar{v}_1, .., \bar{v}_k\}$ are linearly independent, what is a basis for their span, $span(S)$? What is the dimension of $span(S)$?

b) If the $m \times n$ matrix A has rank m, what - explicitly - is its column space, and what - explicitly - is a basis for its column space? Where is a basis of its row space?

c) If the columns of a matrix A are linearly independent, what does the row reduced echelon form of A look like?

d) If $rank(A) = n$, what can we say about solutions of linear systems $A\bar{x} = \bar{b}$ that have A as the matrix of coefficients? Similarly, what can we say about solutions of $A\bar{x} = \bar{b}$ if $rank(A) < n$.

e) Can you create a 5×5 matrix with 4 (but not 5) independent columns and 3 (but not 4) independent rows? Why or why not.

f) If we have a set of column vectors $S = \{\bar{v}_1, .., \bar{v}_k\}$ that spans a subspace V, explain how we computationally go about selecting a subset of the vectors in S that is a basis for V.

2. Given a matrix A and its reduced row echelon form R, answer the following questions:

$$A = \begin{bmatrix} 1 & 2 & -3 & 1 & 3 \\ 1 & 1 & -1 & 1 & 2 \\ 1 & -1 & 3 & 1 & 0 \\ 2 & -1 & 4 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = R$$

a) Find a basis for each of the the spaces $C(A)$, $C(A^T)$ and $N(A)$.

b) Express each column in A as a linear combination of the basis vectors in $C(A)$ and the first and last rows of A as a linear combination of the basis vectors in $C(A^T)$

c) Find a vector in the null space of A with all nonzero components. Explain.

3. If $A = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 2 \\ 2 & -1 & 4 & 1 & -1 \end{bmatrix}$ find the most general solution of $A\bar{x} = \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$.

4. If $A = \begin{bmatrix} 1 & -2 & 2 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$ answer the following without writing down any calculations.

a) What is the rank of A? Why?

b)Provide a basis of the column space of A and of the row space of A. Explain

c) The vectors $(1,1,1,1)$ and $(2,1,-1,-2)$ are clearly in the null space of A. Argue why they must be a basis of the null space.

d) What vectors are in the left null space of A? Why?

5. Let A be an $m \times n$ matrix. Group together the statements below that are equivalent to each other.

a)$rank(A) = m$ b) $rank(A) = n$ c) $rank(A) < m$ d) $rank(A) < n$

e) $A\bar{x} = \bar{b}$ always has a solution f) $A\bar{x} = \bar{0}$ has only one solution

g) $A\bar{x} = \bar{b}$ either has no solutions, or an infinite number, but never only one.

h)The columns of A are independent i)The rows of A are independent

j) The left null space is only the zero vector. k) The null space is only the zero vector.

l) $C(A) = R^m$ m) $C(A^T) = R^n$