Math 441 Exam 3

1. Subspaces of R^n are generally described either as the column space of some given matrix, or the null space of some given matrix.

a) If we define a subspace V as the nullspace of the matrix $A = \begin{bmatrix} -1 & 1 & -2 & 1 \\ 1 & -1 & 1 & -2 \end{bmatrix}$ express V instead as the column space of some matrix B. Explain how/why this works in general.

b) If we define a subspace V as the column space of the matrix $A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \\ -2 & 1 \\ 1 & -2 \end{bmatrix}$ express V instead as the null space of some matrix B. Explain how/why this works in general.

2. Find an orthonormal basis for the subspace V spanned by $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ -1 \end{bmatrix}$

3. The normal equations $A^T(\bar{b} - A\bar{x}) = \bar{0}$ arise how/why in calculating the projection $\bar{p}$ of a vector $\bar{b}$ onto the column space of A. What property do the normal equations express?

4. Use the (amazing) cofactor matrix to calculate the inverse of $\begin{bmatrix} -1 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. If $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$ is an $n \times n$ matrix, and $C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix}$ is its matrix of cofactors, then $\det\begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$ can be expressed as the dot product of ______ with ______.

5. Complete the following by filling in the blanks:

If $A\bar{x} = \bar{b}$ then $\bar{b}$ is in the _________ space of _____.
If $\bar{y}$ is in the orthogonal complement of the column space of $A,$ then $\bar{y}$ is in the _______ space of _____.

If $\bar{x}$ is in the row space of $A,$ then $\bar{x}$ is in the orthogonal complement of the ______ space of _____.

If V is a subspace of R^n of dimension $k,$ then $V^\perp$ has dimension ______. A basis for $V^\perp$ can be calculated from a basis of V as the special solutions of $A\bar{x} = \bar{0}$ where A contains ______________ and the rank of A is _________ .

6. If $A\bar{x} = \bar{b}$ has a solution, then the solution $\bar{x}$ of shortest length is in the row space of A. Why is that true? Explain.

7. Find the values of a and b so that the vector $\bar{p} = a\begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ is as close as possible to the vector $\bar{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. How would you describe the vector $\bar{p}$ using the idea of projection?