Math 441 Exam 4

1. Given $A = \begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix}$

a) Find the eigenvalues and eigenvectors of A

b) Write A in diagonalized form, $A = SDS^{-1}$ where D is a diagonal matrix. Then find a formula for A^k. As $k \to \infty$, every column of $\frac{1}{2^k}A^k$ is a multiple of what vector - what is the significance of this vector?

2. If $A = \begin{bmatrix} 3 & 2 & -1 \\ 6 & 4 & -2 \\ -3 & -2 & 1 \end{bmatrix}$

The rank of A is _____ so one eigenvalue is $\lambda =$____ .

There must be _1 / 2 / 3_____ eigenvectors corresponding to this value of λ because __________.

Those corresponding eigenvectors are:

The characteristic polynomial must have a factor of ____________ because _________ .

The characteristic polynomial of A is $p(\lambda) =$ __________________ (calculate below).

The remaining eigenvalue is _______ and the corresponding eigenvector is ____________. (Show your work below)

3.

If I tell you that the eigenvectors of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ are $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, how can you easily check? What eigenvalues correspond to these eigenvectors? Given that A is symmetric, it should have a complete set of orthonormal eigenvectors? What vectors (explicitly) are those? Write A in the form $A = Q\Lambda Q^T$ where Q is an orthogonal matrix.

4. Explain the following: If $A = SDS^{-1}$ where D is diagonal and S is some nonsingular matrix, then the eigenvalues of A are the columns of S and the eigenvalues of A are the diagonal entries of D.

5. For the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ compute orthonormal vectors $\bar{v}_1$, $\bar{v}_2$ such that the vectors $\bar{u}_1 = A\bar{v}_1$ and $\bar{u}_2 = A\bar{v}_2$ are orthogonal. The vectors $\bar{v}_1$, $\bar{v}_2$ are the eigenvectors of what matrix?