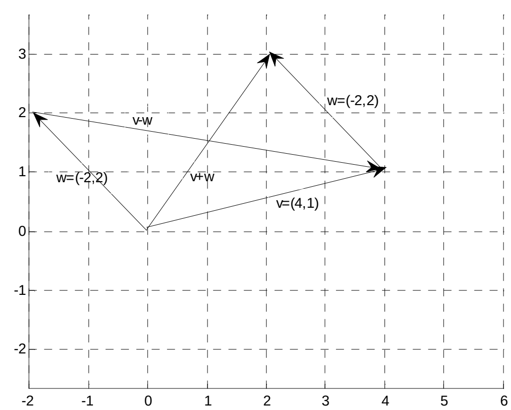
1. a) Notice that the second vector is three times the first, so all linear combinations of the two vectors can be expressed as a scalar multiple of the first vector. So the result is the line $t(1,2,3)$

b) The result is a plane that can be described as $(x,2t,3t)$, a line $(0,2t,3t)$ in the yz plane, with any x anything. You can also describe it as the plane $3y-2z=0$.

c) These linear combinations will generate all vectors in R^3 - you can generate any y,z you want by combining $(0,2,2)$ and $(2,2,3)$ and then "fix up" the x coordinate by adding an appropriate multiple of (2,0,0).

We will have plenty of opportunity later to solve such problems in detail with a failproof algorithm. For now, you're just supposed to think about the possibilities with these simple examples.

2.



3. Adding, we get $2\bar{v}=(6,6)$ and subtracting, $2\bar{w}=(4,-4)$.

4. Looks pretty straightforward.

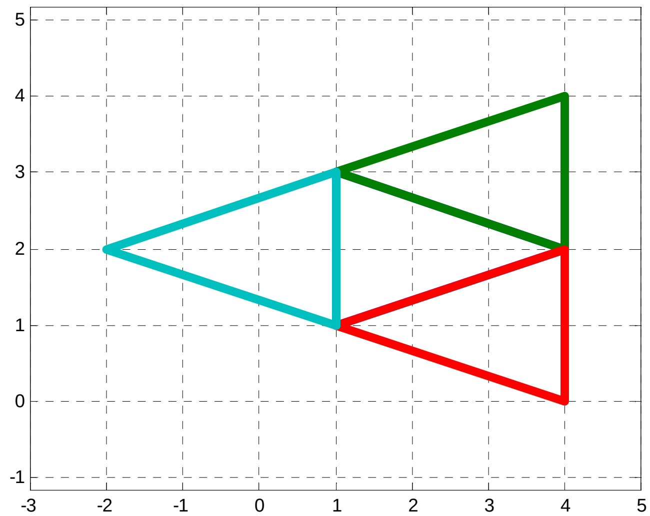
5. Note that $\bar{u}+\bar{v}+\bar{w}=\bar{0}$, so that so that $\bar{w}=-\bar{u}-\bar{v}$. Any linear combination of $\bar{u},\bar{v},\bar{w}$ thus can be re-expressed, by substituting for $\bar{w}$, as a linear combination of just $\bar{u}$ and $\bar{v}$; threfore the result always lies in the plane of $\bar{u}$ and $\bar{v}$.

6. It's pretty clear: that the components of $\bar{v}$ and the components of $\bar{w}$ each add to zero and so any linear combination of $\bar{v}$ and $\bar{w}$ have the same property. In solving $c(1,-2,1)+d(0,1,-1)=(3,3,-6)$ we see that we better have $c=1$ and then $d=5$ will make it work. Note that it's pretty clear that any vector whose components sum to zero can be expressed in this way - c takes care of the first component, then d can be used to fix up the second component, and then the remaining component is restricted and uniquely determined from the first two, because the sum must be zero.

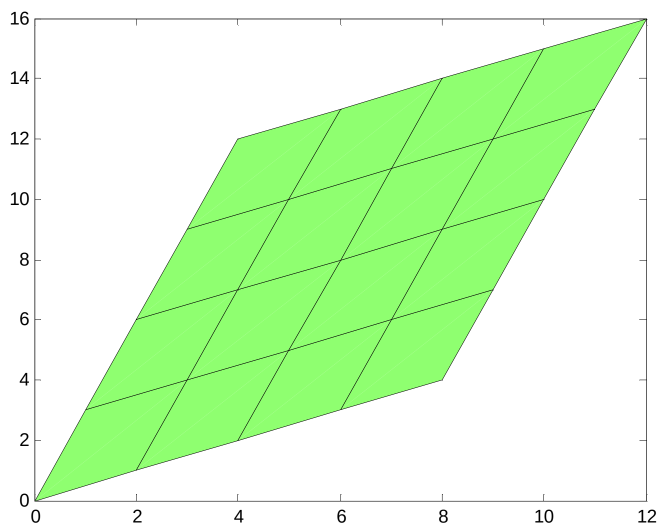
7. The point of this problem as I see it, is to suggest that the values of c and d can be used to construct a new coordinate system, corresponding to two "slanted" axes in the directions of the two given vectors.

8. The other diagonal is $\bar{v}-\bar{w}$ (or else $\bar{w}-\bar{v}$). Adding diagonals gives $2\bar{v}$ (or $2\bar{w}$).

9. Three vertices of a parallelogram, when connected, form a triangle that represents "half" of the parallelogram, where one of the sides of the triangle is a diagonal. There are three possible choices for the diagonal, that leads to three different parallelograms. Here's the picture:



15. We discussed such convex combinations in class. The vector $\frac{3}{4}\bar{v} + \frac{1}{4}\bar{w}$ is $\frac{1}{4}$ of the way from $\bar{v}$ to $\bar{w}$, or at the midpoint between $\bar{u}$ and $\bar{v}$ as shown in the figure.
16. The given points all lie along the line connecting $\bar{v}$ and $\bar{w}$. You might observe, e.g. that $-\bar{v} + 2\bar{w} = \bar{w} + (\bar{w} - \bar{v}) = \bar{v} + 2(\bar{w} - \bar{v})$, which allows you to locate this point.
17. The given combinations are scalar multiples of the vector $\bar{u}$ so they fill out the line through the midpoint of $\bar{v}$ and $\bar{w}$.
18. You can reach with $c\bar{v} + d\bar{w},\ 0 \leq c, d \leq 1$ everything in the parallelogram determined by $\bar{v}$ and $\bar{w}$.
Using the vectors $\bar{v} = (2, 1)$ and $\bar{w} = (1, 3)$ and values $0 \leq c, d \leq 4$ we would get the region below, where integer choices of c, d are at the gridpoints.



19. As noted, you get the cone determined by the two vectors if $c, d \geq 0$.

22. The given combinations are points in the pyramid: If $c + d + e = 1$ you get a point in the base so in general $c\bar{u} + d\bar{v} + e\bar{w} = (c + d + e)\left(\frac{c}{c + d + e}\bar{u} + \frac{d}{c + d + e}\bar{v} + \frac{e}{c + d + e}\bar{w}\right)$. The linear combination on the right puts you in the base, and since $0 \le c + d + e \le 1$ by assumption, the scalar factor in front pulls you in toward the pyramid's apex at the origin. On the other hand $\frac{1}{2}\bar{u} + \frac{1}{2}\bar{v} + \frac{1}{2}\bar{w}$ puts you outside the pyramid because the coefficients sum to 3/2.

23. As long as the three vectors don't all lie in a single plane, any vector can be generated as $c\bar{u} + d\bar{v} + e\bar{w}$. You are suggested to see this by noting that $c\bar{u} + d\bar{v}$ fills out a plane and then adding multiples of $\bar{w}$ will fill out all of R^3 as long as $\bar{w}$ doesn't lie in the same plane.

24. You're supposed to reason geometrically: Combinations of $\bar{u}$ and $\bar{v}$ fill out a plane, combinations of $\bar{v}$ and $\bar{w}$ fill out another plane, these two planes can only intersect in a line which must contain $\bar{v}$ so only multiples of $\bar{v}$ can be in common.

25. If $\bar{u}, \bar{v}, \bar{w}$ are all multiples of a single vector then their combinations fill out only a line. If $\bar{u}$ and $\bar{v}$ fill out a plane and $\bar{w}$ is in that plane then their combinations will only fill out that plane.

26. For the given vector equation we must have, equating components on both sides:

$c + 3d = 14$, $2c + d = 8$