Sec. 1.2
3,4,5,6,8,9,11,12,16,17,21,23

3. $\frac{\bar{v}}{\|\bar{v}\|} = \frac{1}{5}(3,4)$ and $\frac{\bar{w}}{\|\bar{w}\|} = \frac{1}{10}(8,6)$ give unit vectors in the directions of $\bar{v}, \bar{w}$
Then $\cos\theta = \frac{\bar{v} \bullet \bar{w}}{\|\bar{v}\|\,\|\bar{w}\|} = \frac{\bar{v}}{\|\bar{v}\|} \bullet \frac{\bar{w}}{\|\bar{w}\|} = \frac{1}{5}(3,4) \bullet \frac{1}{10}(8,6) = \frac{48}{50} = \frac{24}{25}$, and
$\theta = \cos^{-1}\left(\frac{24}{25}\right) = 0.28379$ (in radians)
Given $\bar{w} = (8,6)$ the vectors $(4,3)$, $(-3,4)$, $(-4,-3)$ make an angle of 0° , 90° , 180°, respectively, with $\bar{w}$.

4. a) $\bar{v} \bullet (-\bar{v}) = -(\bar{v} \bullet \bar{v}) = -\|\bar{v}\|^2 = -1$
b) Expand, the cross-terms cancel: $(\bar{v} + \bar{w}) \bullet (\bar{v} - \bar{w}) = \bar{v} \bullet \bar{v} - \bar{w} \bullet \bar{w} = 0$ (the diagonals of a rhombus meet at right angles)

c) $(\bar{v} + 2\bar{w}) \bullet (\bar{v} - 2\bar{w}) = 1 - 4 = -3$
5. $\bar{u}_1 = \frac{1}{\sqrt{10}}(3,1)$, $\bar{u}_2 = \frac{1}{3}(2,1,2)$
$\bar{U}_1 = (-1,3)$ and $\bar{U}_2 = (1,-2,0)$ or $(0,-2,1)$ are perpendicular to $\bar{u}_1$ and $\bar{u}_2$ respectively. Note that any combination $c(1,-2,0) + d(0,-2,1)$ is perpendicular to $\bar{u}_2$ and that exhausts all the possibilities.
6. a) Any multiple of $(1,2)$ is perpendicular to $(2,-1)$.
b) plane. (This is the plane $x + y + z = 0.$)
c) line. (This is the line through the origin perpendicular to the plane containing the two given vectors)
8. a) Clearly false, since $(1,0,0)$ is perpendicular to $(0,1,0)$ and to $(0,0,1)$ but those latter two vectors are not parallel.
b) True: $\bar{u} \bullet (\bar{v} + 2\bar{w}) = \bar{u} \bullet \bar{v} + 2(\bar{u} \bullet w) = 0$ by assumption so $\bar{u}$ is pependicular to $\bar{v} + 2\bar{w}$. Geometrically, $\bar{u}$ is pependicular to any vector in the plane of $\bar{v}$ and $\bar{w}$ if $\bar{u}$ is perpendicular to $\bar{v}$ and $\bar{w}$.
c) True: $\|\bar{u} - \bar{v}\|^2 = (\bar{u} - \bar{v}) \bullet (\bar{u} - \bar{v}) = \|\bar{u}\|^2 - 2(\bar{u} \bullet \bar{v}) + \|v\|^2 = 1 - 0 + 1 = 2.$ Any two orthogonal vectors that have length 1 are a distance $\sqrt{2}$ apart.
9. If the product of the slopes, $\frac{v_2 w_2}{v_1 w_1} = -1$ then $v_1 w_1 + v_2 w_2 = 0$ or $\bar{v} \bullet \bar{w} = 0$.
11. If $\bar{v} \bullet \bar{w} < 0$ then $\cos\theta < 0$ so $\theta > \pi/2$. In three dimensions, given $\bar{v}$, the vectors $\bar{w}$ satisfying $\bar{v} \bullet \bar{w} < 0$ can be found on the opposite side of the plane that is pependicular to $\bar{v}$.
12. We solve: $(\bar{w} - c\bar{v}) \bullet \bar{v} = 0.$ We distribute through: $\bar{w} \bullet \bar{v} - c\bar{v} \bullet \bar{v} = 0$, $c = \frac{\bar{w} \bullet \bar{v}}{\bar{v} \bullet \bar{v}}$ and $\bar{p} = c\bar{v} = \left(\frac{\bar{w} \bullet \bar{v}}{\bar{v} \bullet \bar{v}}\right)\bar{v}$ is then referred to as the "projection" of $\bar{w}$ on $\bar{v}$.
16. $\|\bar{v}\| = \|(1,1,..,1)\| = \sqrt{9} = 3.$ So $\bar{u} = \frac{1}{3}(1,1,..,1)$ is a unit vector in the direction of $\bar{v}$.
$\bar{w} = \frac{1}{\sqrt{2}}(1,-1,0,..,0)$ is perpendicular to $\bar{v}$, with length 1.
17. If $\bar{v} = (1,0,-1)$ then $\cos\alpha = \frac{\bar{v} \bullet \bar{i}}{\|\bar{v}\|} = \frac{1}{\sqrt{2}}$, $\cos\beta = \frac{\bar{v} \bullet \bar{j}}{\|\bar{v}\|} = 0$, $\cos\gamma = \frac{\bar{v} \bullet k}{\|\bar{v}\|} = \frac{-1}{\sqrt{2}}$ and the sum of the squares is 1 as advertised.
21. Using the suggestion
$\|\bar{v} + \bar{w}\|^2 = \|\bar{v}\|^2 + 2\bar{v} \bullet \bar{w} + \|\bar{w}\|^2 \le \|\bar{v}\|^2 + 2\|\bar{v}\|\,\|\bar{w}\| + \|\bar{w}\|^2 = (\|\bar{v}\| + \|w\|)^2$ and taking the

square roots on both sides, we have the triangle inequality $\|\bar{v}+\bar{w}\| \leq \|\bar{v}\| + \|\bar{w}\|$

23. $\cos\beta = \dfrac{w_1}{\|\bar{w}\|}$, $\sin\beta = \dfrac{w_2}{\|\bar{w}\|}$. Now

$$\cos\theta = \cos(\beta-\alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha = \frac{w_1v_1}{\|\bar{w}\|\,\|\bar{v}\|} + \frac{w_2v_2}{\|\bar{w}\|\,\|\bar{v}\|} = \frac{\bar{w}\bullet\bar{v}}{\|\bar{w}\|\,\|\bar{v}\|}.$$

(Actually I think it's more impressive going the other way - deriving the trig formula from the dot product formula for $\cos\theta$).