MATH 251.009 (5:00pm) NAME (print): ____________________
Instr. K. Ciesielski
Spring 2013

Quiz # 1

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

Ex. 1. Solve each of the following systems of linear equations by Gauss elimination. (You must use augmented matrix approach.) If the system inconsistent, give a reason for it explain the meaning of it. If it is consistent, express its solution in the vertical vector form such as

$$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \text{ or } \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 5 \\ 11 \end{pmatrix} \text{ or } \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 5 \\ 11 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}.$$

(a) $\begin{cases} 2x + 2y - 6t = 4 \\ -3x - 3y + 9t = -6 \end{cases}$ (**4pts**)

Augmented matrix and its elimination are as follows:

$$\left[\begin{array}{rrrr} 2 & 2 & -6 & 4 \\ -3 & -3 & 9 & -6 \end{array}\right] \begin{array}{l} \times(1/2) \\ \times(1/3) \end{array} \to \left[\begin{array}{rrrr} 1 & 1 & -3 & 2 \\ -1 & -1 & 3 & -2 \end{array}\right] \begin{array}{l} \\ +R_1 \end{array} \to \left[\begin{array}{rrrr} 1 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

This reduces to the single equation: $x+y-3t = 2$, and we can express x in terms of y and t via $x = 2-y+3t$. So, x is expressed in a general form $x = a_1+b_1y+c_1t$. Similarly, we express y and t in this format: $y = a_2+b_2y+c_2t$ as $y = y$ and $t = a_3+b_3y+c_3t$ as $t = t$.

Now, using the general format $\begin{bmatrix} x \\ y \\ t \end{bmatrix} = \begin{bmatrix} a_1 + b_1y + c_1t \\ a_2 + b_2y + c_2t \\ a_3 + b_3y + c_3t \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_2 \end{bmatrix} + y \begin{bmatrix} b_1 \\ b_2 \\ b_2 \end{bmatrix} + t \begin{bmatrix} c_1 \\ c_2 \\ c_2 \end{bmatrix}$

we get

$$\begin{bmatrix} x \\ y \\ t \end{bmatrix} = \begin{bmatrix} 2 - y + 3t \\ y \\ t \end{bmatrix} = \begin{bmatrix} 2 - 1y + 3t \\ 0 + 1y + 0t \\ 0 + 0y + 1t \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}.$$

Answer:

$$\begin{bmatrix} x \\ y \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}.$$

(b) $\begin{cases} a + 3b = 2 \\ 4a + 12b = 8 \\ -3a - 9b = -6 \end{cases}$ (**3pts**)

Augmented matrix and its elimination are as follows:

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 12 & 8 \\ -3 & -9 & -6 \end{bmatrix} \begin{matrix} \\ \times(1/4) \\ \times(1/3) \end{matrix} \rightarrow \begin{bmatrix} 1 & 3 & 2 \\ 1 & 3 & 2 \\ -1 & -3 & -2 \end{bmatrix} \begin{matrix} \\ -R_1 \\ +R_1 \end{matrix} \rightarrow \begin{bmatrix} 1 & 3 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So, this reduces to the single equation: $a + 3b = 2$, and we can express a in terms of b via $a = 2 - 3b$. To find $\begin{bmatrix} a \\ b \end{bmatrix}$ in terms of b we can put simply $b = b$. This gives

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2-3b \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} -3b \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + b \begin{bmatrix} -3 \\ 1 \end{bmatrix}.$$

Answer:

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + b \begin{bmatrix} -3 \\ 1 \end{bmatrix}.$$

Ex. 2. Compute

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 6 & 4 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 2 & 3 \end{pmatrix} \quad (\textbf{3pts}) \text{ Answer: } \begin{pmatrix} 3 & 2 \\ 7 & 5 \\ 7 & 8 \end{pmatrix}$$