Advanced Analysis of Algorithms - Final

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1 Instructions

1. You are required to turn in the exam by 1 pm.
2. You are permitted to use class notes and the course textbook.
3. Each question is worth 4 points.

2 Problems

1. The Maximum Subarray problem is defined as follows: Given an array $\mathbf{A}[1 \cdot\cdot n]$ of n integers (with at least one positive element) find a contiguous sub-array within $\mathbf{A}$ which has the largest sum.

   For instance, in the array $\mathbf{A}$ defined by $A[1] = -2$, $A[2] = 1$, $A[3] = -3$, $A[4] = 4$, $A[5] = -1$, $A[6] = 2$, $A[7] = 1$, $A[8] = -5$, $A[9] = 4$, the contiguous subarray with the largest sum is $A[4 \cdot\cdot 7]$ with sum $4 + (-1) + 2 + 1 = 6$.

   Design a Divide-and-Conquer algorithm for the Maximum Subarray problem and analyze its running time.

2. As discussed in class, a *matching* in a graph $\mathbf{G} = \langle V, E \rangle$ is a collection of vertex-disjoint edges. The size of a matching is the number of edges in the disjoint collection. A *perfect matching* is a matching of size $\frac{|V|}{2}$. Observe that in a perfect matching, every vertex is matched to another vertex. Likewise, if a graph with an odd number of vertices cannot have a perfect matching. Design a *linear-time* algorithm to check whether $\mathbf{G}$ has a perfect matching, under the assumption that $\mathbf{G}$ has no cycles.

3. In class, we defined the 2 partition problem as follows: Let $S = \{a_1, a_2, \ldots, a_n\}$ denote a set of n integers. Does there exist a set $A \subseteq S$, such that such that $\sum_{a_i \in A} a_i = \sum_{a_j \in S \setminus A} a_j$. In other words, the sum of the elements in A is equal to the sum of the elements not in A. Design a dynamic programming algorithm for the 2-partition problem that runs in time $O(n \cdot N)$, where $N = \sum_{a_i \in S} a_i$. Does your algorithm establish that this problem is in the class $\mathbf{P}$?

4. (a) Let A be an **NP-complete** set and let B be a set in **P**. Assume that the $A \cap B = \emptyset$. What is the complexity of the set $A \cup B$? If $A \cap B \neq \emptyset$, what can you say about the complexity of $A \cup B$?

   (b) Let A and B be two sets in $\mathbf{NP}$. Establish that the sets $A \cap B$ and A^* are also in $\mathbf{NP}$.

5. The Minimal SAT problem is defined as follows: Given a 3CNF formula ϕ, defined over the variables $\{x_1, x_2, \ldots, x_n\}$ and the clauses C_1, C_2, ..., C_m, is there a satisfying assignment for ϕ, such that exactly one literal in each clause is set to **true**? Prove that the Minimal SAT problem is **NP-complete**. (*Hint: 3SAT or 4SAT.*)