Mathematical Induction

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22 August, 2013

Outline

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Induction

Motivation

Reaching arbitrary rungs of a ladder.

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Note

Can only be applied to a well-ordered domain, where the concept of “next” is unambiguous, e.g. integers.

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$P(n)$ is **true**, for all positive integers n.

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(i) *Showing that $P(1)$ is* **true** *is called the basis step.*

(ii) *Assuming that $P(k)$ is* **true***, in order to show that $P(k+1)$ is* **true** *is called the inductive hypothesis.*

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Show that the sum of the first n integers is $\frac{n \cdot (n+1)}{2}$.

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Thus, $LHS = RHS$ and $P(1)$ is true. □

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Let us assume that $P(k)$ is true, i.e., assume that

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Since, LHS=RHS, we have shown that $P(k) \rightarrow P(k+1)$.

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Applying the first principle of mathematical induction, we conclude that the conjecture is true. $\square$

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Show that the sum of the squares of the first n integers is $\frac{n \cdot (n+1) \cdot (2n+1)}{6}$,

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&= \frac{k \cdot (k+1) \cdot (2k+1)}{6} + (k+1)^2, \quad \text{using the inductive hypothesis} \\
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&= \frac{k+1}{6}(2k^2 + k + 6k + 6) \\
&= \frac{k+1}{6}(2k^2 + 7k + 6)
\end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i^2 = \frac{k \cdot (k+1) \cdot (2k+1)}{6}.$$

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$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i^2 \\
&= 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 \\
&= (1^2 + 2^2 + 3^2 + \ldots + k^2) + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2k+1)}{6} + (k+1)^2, \quad \text{using the inductive hypothesis} \\
&= \frac{k+1}{6}(k \cdot (2k+1) + 6 \cdot (k+1)) \\
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$\square$

Induction proof (contd.)

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Since, *LHS*=*RHS*, we have shown that $P(k) \rightarrow P(k+1)$.

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Applying the first principle of mathematical induction, we conclude that the conjecture is true. □

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Example

Show that the sum of the first n odd integers is n^2, i.e., show that $\sum_{i=1}^{n}(2i-1) = n^2$.

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LHS &= \sum_{i=1}^{1}(2i-1) \\
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&= 1 \\
RHS &= 1^2 \\
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Thus, $LHS = RHS$ and $P(1)$ is true. □

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

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Since $LHS = RHS$, we have shown that $P(k) \rightarrow P(k+1)$.

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\end{aligned}$$

Since $LHS = RHS$, we have shown that $P(k) \rightarrow P(k+1)$. Applying the first principle of mathematical induction, we conclude that the conjecture is true. $\square$

One Final Example

Example

Show that $7^n - 5^n$ is always an even number for $n \geq 0$, i.e., show that $2 \mid (7^n - 5^n)$, $\forall n \geq 0$.

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Show that $7^n - 5^n$ is always an even number for $n \geq 0$, i.e., show that $2 \mid (7^n - 5^n)$, $\forall n \geq 0$.

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BASIS ($P(0)$):

$$\begin{aligned} LHS &= 7^0 - 5^0 \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

Since the LHS is even, we have proven the basis $P(0)$. □

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k.

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Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m.

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$$7^{k+1} - 5^{k+1} = 7 \cdot 7^k - 5 \cdot 5^k$$

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$$\begin{aligned} 7^{k+1} - 5^{k+1} &= 7 \cdot 7^k - 5 \cdot 5^k \\ &= 7 \cdot (2m + 5^k) - 5 \cdot 5^k, \text{ using the inductive hypothesis} \\ &= 14m + 7 \cdot 5^k - 5 \cdot 5^k \end{aligned}$$

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We have thus shown that $P(k) \to P(k+1)$. Applying the first principle of mathematical induction, we conclude that the conjecture is true. □

Second Principle of Induction

Note

Also called Strong Induction. Is necessary, when the first principle does not help us.

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Show that every number greater than or equal to 8 can be expressed in the form $(5 \cdot a + 3 \cdot b)$, for suitably chosen a and b.

Proof.

The conjecture is clearly true for 8, 9 and 10. Assume that the conjecture holds for all r, $8 \leq r \leq k$. Consider the integer $k + 1$. Without loss of generality, we assume that $(k + 1) \geq 11$. Observe that $(k + 1) - 3 = (k - 2)$ is at least 8 and less than k. As per the inductive hypothesis, $(k - 2)$ can be expressed in the form $(3 \cdot a + 5 \cdot b)$, for suitably chosen a and b. It follows that $(k + 1) = 3 \cdot (a + 1) + 5 \cdot b$, can also be so expressed. Applying the second principle of mathematical induction, we conclude that the conjecture is true. □