Recurrence Relations

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Outline

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1. Solving Recurrences
2. The Master Method

Sample Recurrences

Examples

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Examples

(i)

$$\begin{aligned} S(1) &= 2 \\ S(n) &= 2 \cdot S(n-1), \ n \geq 2. \end{aligned}$$

Sample Recurrences

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$$\begin{aligned} S(1) &= 2 \\ S(n) &= 2 \cdot S(n-1), \; n \geq 2. \end{aligned}$$

(ii)

$$\begin{aligned} T(1) &= 1 \\ T(n) &= T(n-1) + 3, \; n \geq 2. \end{aligned}$$

Sample Recurrences

Examples

(i)

$$\begin{aligned} S(1) &= 2 \\ S(n) &= 2 \cdot S(n-1),\ n \geq 2. \end{aligned}$$

(ii)

$$\begin{aligned} T(1) &= 1 \\ T(n) &= T(n-1) + 3,\ n \geq 2. \end{aligned}$$

(iii)

$$\begin{aligned} F(1) &= 1 \\ F(2) &= 1 \\ F(n) &= F(n-1) + F(n-2),\ n \geq 3 \end{aligned}$$

Solving recurrences

Two methods

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(i) Expand-Guess-Verify (EGV).

Solving recurrences

Two methods

(i) Expand-Guess-Verify (EGV).
(ii) Formula.

EGV

Example

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\begin{aligned}
S(1) &= 2 \\
S(n) &= 2 \cdot S(n-1), \ \ n \geq 2.
\end{aligned}
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EGV

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(i) Expand: $S(1) = 2$,

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(i) Expand: $S(1) = 2$, $S(2) = 2 \cdot 2 = 4$,

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(ii) Guess: $S(n) = 2^n$

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Since LHS=RHS, the basis is proven.

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Since LHS=RHS, the basis is proven.
INDUCTIVE STEP: Assume that $S(k) = 2^k$.

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$$LHS = 2$$
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INDUCTIVE STEP: Assume that $S(k) = 2^k$. We need to show that $S(k+1) = 2^{k+1}$.

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$$S(k+1) = 2 \cdot S(k), \quad \text{by definition}$$

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INDUCTIVE STEP: Assume that $S(k) = 2^k$. We need to show that $S(k+1) = 2^{k+1}$. Observe that,

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INDUCTIVE STEP: Assume that $S(k) = 2^k$. We need to show that $S(k+1) = 2^{k+1}$. Observe that,

$$\begin{aligned} S(k+1) &= 2 \cdot S(k), \quad \text{by definition} \\ &= 2 \cdot 2^k, \quad \text{by inductive hypothesis} \\ &= 2^{k+1}! \end{aligned}$$

Applying the first principle of mathematical induction, we conclude that $S(n) = 2^n$.

EGV (contd.)

Example

Solve the recurrence:

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$$\begin{aligned} T(1) &= 1 \\ T(n) &= T(n-1)+3,\ n \geq 2. \end{aligned}$$

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Solve the recurrence:

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(i) Expand: $T(1) = 1$, $T(2) = T(1) + 3 = 4$,

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Solve the recurrence:

$$\begin{aligned} T(1) &= 1 \\ T(n) &= T(n-1) + 3, \; n \geq 2. \end{aligned}$$

(i) Expand: $T(1) = 1$, $T(2) = T(1) + 3 = 4$, $T(3) = 3 + T(n-2) = 7$,

EGV (contd.)

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(ii) Guess: $T(n) = 3 \cdot n - 2$.

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(iii) Verify: Somebody from class!

Formula approach

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Definition

A general linear recurrence has the form:

$$S(n) = f_1(n) \cdot S(n-1) + f_2(n) \cdot S(n-2) + \ldots f_k(n) \cdot S(n-k) + g(n)$$

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The above formula is called linear, *because the* $S()$ *terms occur only in the first power.*

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For example, $S(n) = c \cdot S(n-1) + g(n)$*. The recurrence is called homogeneous, if* $g(n) = 0$*, for all* n*.*

Formula for Linear first-order recurrence

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Formula for Linear first-order recurrence

$$S(1) = k_0$$

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$$S(n) = f_1(n) \cdot S(n-1) + f_2(n) \cdot S(n-2) + \ldots f_k(n) \cdot S(n-k) + g(n)$$

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The above formula is called linear*, because the* $S()$ *terms occur only in the first power.*
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For example, $S(n) = c \cdot S(n-1) + g(n)$*. The recurrence is called homogeneous, if* $g(n) = 0$*, for all n.*

Formula for Linear first-order recurrence

$$\begin{aligned} S(1) &= k_0 \\ S(n) &= c \cdot S(n-1) + g(n) \end{aligned}$$

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A general linear recurrence has the form:

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For example, $S(n) = c \cdot S(n-1) + g(n)$*. The recurrence is called homogeneous, if* $g(n) = 0$*, for all* n*.*

Formula for Linear first-order recurrence

$$\begin{aligned} S(1) &= k_0 \\ S(n) &= c \cdot S(n-1) + g(n) \end{aligned}$$

$\Rightarrow S(n) = c^{n-1} \cdot k_0 + \sum_{i=2}^{n} c^{n-i} \cdot g(i)$.

Linear first-order recurrence

Example

$$\begin{aligned} S(1) &= 2 \\ S(n) &= 2 \cdot S(n-1),\ n \geq 2. \end{aligned}$$

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As per the formula, $k_0 =$

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S(1) &= 2 \\
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Linear first-order recurrence

Example

$$S(1) = 2$$
$$S(n) = 2 \cdot S(n-1), \; n \geq 2.$$

As per the formula, $k_0 = 2$, $g(n) = 0$ and $c =$

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As per the formula, $k_0 = 2$, $g(n) = 0$ and $c = 2$. Thus,

$$S(n) = 2^{n-1} \cdot 2 + \sum_{i=2}^{n} 2^{n-i} \cdot 0$$

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As per the formula, $k_0 = 2$, $g(n) = 0$ and $c = 2$. Thus,

$$\begin{aligned} S(n) &= 2^{n-1} \cdot 2 + \sum_{i=2}^{n} 2^{n-i} \cdot 0 \\ &= 2^n \end{aligned}$$

Another Example

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Solve the recurrence:

$$\begin{aligned} S(1) &= 4 \\ S(n) &= 2 \cdot S(n-1) + 3, \;\; n \geq 2. \end{aligned}$$

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As per the formula, $k_0 = 4$, $g(n) = 3$ and $c = 2$. Thus,

$$S(n) = 2^{n-1} \cdot 4 + \sum_{i=2}^{n} 2^{n-i} \cdot 3$$

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As per the formula, $k_0 = 4$, $g(n) = 3$ and $c = 2$. Thus,

$$\begin{aligned} S(n) &= 2^{n-1} \cdot 4 + \sum_{i=2}^{n} 2^{n-i} \cdot 3 \\ &= 2^{n+1} + 3 \sum_{i=2}^{n} 2^{n-i} \end{aligned}$$

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$$\begin{aligned} S(n) &= 2^{n-1} \cdot 4 + \sum_{i=2}^{n} 2^{n-i} \cdot 3 \\ &= 2^{n+1} + 3 \sum_{i=2}^{n} 2^{n-i} \\ &= 2^{n+1} + 3 \cdot [2^{n-2} + 2^{n-3} + \ldots + 2^0] \end{aligned}$$

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As per the formula, $k_0 = 4$, $g(n) = 3$ and $c = 2$. Thus,

$$\begin{aligned} S(n) &= 2^{n-1} \cdot 4 + \sum_{i=2}^{n} 2^{n-i} \cdot 3 \\ &= 2^{n+1} + 3 \sum_{i=2}^{n} 2^{n-i} \\ &= 2^{n+1} + 3 \cdot [2^{n-2} + 2^{n-3} + \ldots + 2^{0}] \\ &= 2^{n+1} + 3 \cdot [2^{n-1} - 1]. \end{aligned}$$

Second Order homogeneous Linear Recurrence with constant coefficients

Formula

Second Order homogeneous Linear Recurrence with constant coefficients

Formula

(i) Form: $S(n) = c_1 \cdot S(n-1) + c_2 \cdot S(n-2)$, subject to some initial conditions.

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Examples of second order recurrences

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Solve the recurrence relation

$$\begin{aligned} T(1) &= 5 \\ T(2) &= 13 \\ T(n) &= 6 \cdot T(n-1) - 5 \cdot T(n-2),\ n \geq 3 \end{aligned}$$

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(iii) Accordingly, the solution is $T(n) = 3 \cdot 1^{n-1} + 2 \cdot 5^{n-1} = 3 + 2 \cdot 5^{n-1}$.

□

One More Example

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Solve the recurrence relation:

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\begin{aligned}
S(1) &= 1 \\
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Suppose your recurrence has the following form:

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(ii) If there is a constant $k \geq 0$, such that $f(n) \in \Theta(n^r \cdot \log^k n)$, then $T(n) \in \Theta(n^r \log^{k+1} n)$.

(iii) If there are small constants $\epsilon > 0$ and $\delta > 1$, such that $f(n) \in \Omega(n^{r+\epsilon})$, and $a \cdot f(\frac{n}{b}) \leq \delta \cdot f(n)$, for $n \geq d$, then $T(n) \in \Theta(f(n))$.

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5. $T(n) = 2 \cdot T(\sqrt{n}) + \log n$.