Advanced Analysis of Algorithms

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1. Floor, ceiling, Σ, Π, ∞.
2. Functions, mapping, domain, range, codomain, injective, surjective, graphing.
3. Mathematical induction. $\sum_{i=1}^{n} i, i^2, i^3$. Basis need not be 1 or 0. $\sum_{r=0}^{n} r^i = \frac{r^{n+1}-1}{r-1}$. Slides.
4. Conjecture, Lemma, Theorem.
5. Logarithms - definition, base, exponent, Common, natural, Properties:
   (a) $\log_a 1 = 0$.
   (b) $\log_a a = 1$.
   (c) $a^{\log_a x} = x$.
   (d) product and quotient rules.
   (e) exponent rule. $x^{\log_a y} = y^{\log_a x}$.
   (f) $\log_a x = \frac{\log_b x}{\log_b a}$.
6. Monotonic increasing and decreasing functions.
7. If f is monotonically increasing,
$$\int_{m-1}^{n} f(x) \cdot dx \leq \sum_{k=m}^{n} f(k) \leq \int_{m}^{n+1} f(x) \cdot dx$$
8. If f is monotonically decreasing,
$$\int_{m}^{n+1} f(x) \cdot dx \leq \sum_{k=m}^{n} f(k) \leq \int_{m-1}^{n} f(x) \cdot dx$$
9. Derive a tight bound on H_n. $f(k) = \frac{1}{k}$.
$\sum_{k=1}^{n} \frac{1}{k} \geq \int_{1}^{n+1} \frac{dx}{x} = \ln(n+1)$.
$\sum_{k=1}^{n} \frac{1}{k} = 1 + \sum_{k=2}^{n} \frac{1}{k} \leq 1 + \int_{1}^{n} \frac{dx}{x} = \ln n + 1$.
10. Sets - Definition, members, equality of sets, subset, proper subset, union, intersection, difference, empty set, Universal set.
11. Permutations, Combinations and the Binomial Theorem, Number of subsets of a set of size n. Use slides.