CS 591 - Combinatorial Optimization

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1 General Information

1. Semester - Spring 2013.
2. Meeting Times: M-W, 12 noon - 1:15 am.
   Location: 215 ESB-E.
3. Contact Information: 749 ESB, ksmani@csee.wvu.edu.
4. Office Hours: By appointment.
5. Prerequisites: Exposure to probability and algorithm design.
6. Textbook - [Lee04] is the main text, although [KV10] and [Sch87] are strongly recommended for supplementary reading.
7. URL - `http://www.csee.wvu.edu/~ksmani/courses/sp13/combopt/combopt.html`.
8. Assessment:
   (a) Homeworks (4) - There will be four homework assignments; each assignment is worth 20% of your grade. Table (1) details the homework schedule.

Assignment Date	Submission Date
01/31	02/07
02/28	03/07
04/03	04/10
05/02	05/09

Table 1: Homework Schedule

   (b) Presentation (1) - You will be required to present one topic which will be decided through discussions with the instructor. The presentation is worth 20 points.
   (c) Final - There will be no final exam.
   (d) A maximum of 5 bonus points will be awarded for class performance.

9. Grade Boundaries:

Grade	Boundary
A	80 and up
B	$65 - 79$
C	$50 - 64$
D	$45 - 49$
F	$0 - 44$

Table 2: Grade Boundaries

10. Grading policy - If you have any questions about the grading, you must contact the intructor within two days of your paper being returned.

11. Makeup Policy - If for some reason, you are unable to attend a test or an exam, please meet me at the earliest and I will set an alternate date.

12. **Course Objectives** - The objectives of this course are as follows:

    (a) Introducing the fundamentals of combinatorial optimization.

    (b) Learning how to model problems using mathematical programs.

    (c) Understanding polyhedral analysis.

    (d) Introducing the fundamentals of matroid theory.

    (e) Introducing submodular function optimization.

13. **Learning Outcomes** - Upon successful completion of this course, students will be able to:

    (a) Model problems using linear and integer programs.

    (b) Develop efficient approaches to network optimization problems in **P**.

    (c) Observe the existence of matroid structures in problems, if they exist.

    (d) Develop branch-and-bound approaches to “hard” problems.

    (e) Design algorithms for cut and flow variants.

    (f) Design algorithms for submodular function optimization.

2 Syllabus Sketch and Weekly Schedule

2.1 Rudiments

2.1.1 Polytopes and Linear Programming

Finite systems of linear inequalities, Linear programming duality, Basic solutions and the Primal Simplex method, Sensitivity analysis, Polytopes, Lagrangian relaxation, The Dual Simplex method, Totally Unimodular matrices, Graphs and Digraphs.

The above material is covered in Chapter 0 of [Lee04].

2.2 Elementary Analysis

2.2.1 Matroids and the Greedy Algorithm

Independence axioms and examples of matroids, Circuit properties, Representations, The Greedy Algorithm, Rank properties, Duality, The Matroid Polytope.

2.2.2 Minimum-weight Dipaths

Negative weight cycles, All-Pairs Minimum-weight Dipaths, Non-negative weights, No Dicycles and Knapsack programs.

2.2.3 Matroid Intersection

Applications of matroid intersection, An efficient cardinality matroid intersection algorithm and its consequences, An efficient maximum-weight matroid intersection algorithm, The matroid intersection polytope.

The above material is covered in Chapters 1, 2 and 3 of [Lee04].

2.3 Advanced Analysis

2.3.1 Matching

Augmenting paths and matroids, The Matching polytope, Duality and a Maximum-Cardinality Matching Algorithm, Kuhn's algorithm for the Assignment problem, Applications of Weighted Matching.

2.3.2 Flows and Cuts

Source-Sink Flows and Cuts, An Efficient Maximum Flow algorithm and consequences, Undirected cuts.

The above material is covered in Chapters 4 and 5 of [Lee04].

2.4 Advanced Tools

2.4.1 Cutting Planes

The generic Cutting-Plane method, Chvátal-Gomory cutting planes, Gomory cutting planes, Tightening a constraint, Constraint generation for combinatorial optimization problems.

2.4.2 Branch & Bound

Branch & Bound using linear programming relaxation, Knapsack programs and group relaxation, Branch & Bound for optimal-weighted Hamiltoninan tour, Maximum-Entropy sampling and Branch & Bound.

2.4.3 Optimizing Submodular Functions

Minimizing submodular functions, Minimizing submodular functions over odd sets, Maximizing submodular functions.

The above material is covered in Chapters 6, 7 and 8 of [Lee04].

I would like to reiterate that this is a sketch of the topics that we will be covering. For various reasons, I may choose to drop a mentioned topic or cover a new topic. In such cases, advance notice will be given.

3 Academic Integrity Statement

The integrity of the classes offered by any academic institution solidifies the foundation of its mission and cannot be sacrificed to expediency, ignorance, or blatant fraud. Therefore, I will enforce rigorous standards of academic integrity in all aspects and assignments of this course. For the detailed policy of West Virginia University regarding the definitions of acts considered to fall under academic dishonesty and possible ensuing sanctions, please see the Student Conduct Code at `http://www.arc.wvu.edu/admissions/integrity.html`. Should you have any questions about possibly improper research citations or references, or any other activity that may be interpreted as an attempt at academic dishonesty, please see me before the assignment is due to discuss the matter.

4 Social Justice Statement

West Virginia University is committed to social justice. I concur with that commitment and expect to foster a nurturing learning environement, based upon open communication, mutual respect and non-discrimination. Our University does not discriminate on the basis of race, sex, age, disability, veteran status, religon, sexual orientation, color or national origin. Any suggestions to further such a positive and open environment in this class will be appreciated and given serious consideration. If you are a person with a disability and anticipate needing any type of accommodation in order to participate in this class, please advise me of the same and make appropriate arrangments with Disability Services ($293 - 6700$).

If you feel that you are being treated inappropriately or unfairly in any way, please feel free to bring your concerns to my attention; rest assured that doing so will not prejudice the grading process. In return, I expect you to behave professionally and ethically.

References

[KV10] B. Korte and J. Vygen. *Combinatorial Optimization*. Number 21 in Algorithms and Combinatorics. Springer-Verlag, New York, 4^{th} edition, 2010.

[Lee04] Jon Lee. *A First Course in Combinatorial Optimization*. Cambridge University Press, 1^{st} edition, 2004.

[Sch87] Alexander Schrijver. *Theory of Linear and Integer Programming*. John Wiley and Sons, New York, 1987.