Discrete Mathematics 2 - Homework III

K. Subramani
LCSEE,
West Virginia University,
Morgantown, WV
{ksmani@csee.wvu.edu}

1 Instructions

1. The homework is due on April 10, in class.
2. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.
3. Please show all the steps in your proof, using explicit justifications.
4. The work must be entirely your own. You are expressly **prohibited** from consulting with colleagues or the internet (with the exception of the material on the course website and the laws of trigonometry.)

2 Problems

1. Let $f^{(i)}(a)$ denote the application of the function f to the argument a, i times. For instance $f^{(2)}(a) = f(f(a))$, $f^{(3)}(a) = f(f(f(a)))$ and so on. Is the following formula valid in the theory of equality:
$$[\ f^{(3)}(a) = f^{(2)}(a) \wedge f^{(4)}(a) = a\ ] \rightarrow [\ f(a) = a\ ].$$
2. Consider the following formula:
$$F : [\ (a[i] \geq 1) \wedge (a[i] + x \leq 2) \wedge (x > 0) \wedge (x = i)\ ] \rightarrow [\ a\langle x \lhd 2\rangle[i] = 1\ ]$$
Is F valid in the theory $(T_A \cup T_{\mathbb{Z}})$?
3. Argue that
$$(\cos\theta + i \cdot \sin\theta)^n = \cos(n \cdot \theta) + i \cdot \sin(n \cdot \theta),\ \forall n \geq 1.$$
In the above expression, i is the square-root of -1, i.e., $i = \sqrt{-1}$.
4. Prove that the following formulae in T^{+}_{cons} are valid:
   (a) $(\forall u)(\forall v)\ [\ flat(u) \wedge flat(v)\ ] \rightarrow flat(\text{concat}(u, v))$.
   (b) $(\forall u)\ flat(u) \rightarrow flat(rvs(u))$.
5. Consider the Fibonacci sequence defined by:
$$\begin{aligned} F(1) &= 1 \\ F(2) &= 1 \\ F(n) &= F(n-1) + F(n-2),\ n > 2 \end{aligned}$$
Show that $F(n) = \frac{p^n - q^n}{p - q}$, where $p = \frac{1+\sqrt{5}}{2}$ and $q = 1 - p$.