Final - Due Tuesday 12/12/2000

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1 Asymptotic Notation

1. Is $n = O(0.2831n)$? (5 pts.)
2. What is the asymptotic relationship between n^2 and $n. \log^5 n$? (5 pts.)

2 Divide and Conquer

1. Let $A[1 \ldots n]$ be an array of n *distinct* numbers. A pair (i, j) is called an *inversion pair* if $i < j$ and $A[i] > A[j]$. For example, the array $A = [2, 3, 8, 6, 1]$ has 5 inversion pairs, viz. $(1, 5)$ (since $A[1] = 2 >$A[5]=1), $(2, 5), (3, 4), (3, 5)$ and $(4, 5)$. Design a Divide and Conquer algorithm that determines the *number* of inversion pairs in an array of size n. Provide an analysis of the running time. (20 pts.)

3 Greedy

1. Consider the problem of making change for n cents using the least number of coins.
   (a) Describe a greedy procedure to make change consisting of quarters, dimes, nickels and pennies. Prove that your algorithm is correct. (15 pts.)
   (b) Give a set of coin denominations for which the greedy strategy fails (10 pts.)

4 Dynamic Programming

1. Assume that n programs are to be stored on two tapes T_1 and T_2. Let l_i be the length of tape needed to store the i^{th} program. Assume that $\sum_{i=1}^{n} l_i \leq L$, where L is the length of each tape. A program may be stored on on either T_1 or T_2. If S_1 is the set of programs on T_1, then the worst-case access time for a program is proportional to $\max\{\sum_{i \in S_1} l_i, \sum_{i \notin S_1} l_i\}$. Formulate a dynamic programming algorithm to determine the worst case access time of an *optimal* assignment. Analyze the running time of your algorithm. (25 pts.)

5 Graph Algorithms

1. In class, we studied two algorithms for the All-Pairs shortest path problem. In both cases, we assumed that a negative weight cycle did not exists in the graph. Modify either algorithm to provide a test to detect negative weight cycles. (*Note: Do not use Bellman-Ford; it works only for single-source shortest paths.*) (20 pts.)