Homework II - Solutions

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1. There are 2 key observations to make:

   (a) Any optimal solution on k lines consists of p (say) words on the first line and the remaining $n-p$ words on the remaining $k-1$ lines.

   (b) If all the words fit on one line, it is sub-optimal to break up the words into two or more lines.

   These two simple observations are enough to provide you with the recurrence relation. Let $s[i,j]$ denote the packing cost of packing words w_i through w_j *in one line*. The following recurrences are immediate

$$\begin{aligned} s[i,j] &= (M-j+i-\sum_{t=i}^{j} l_t)^3,\ if\ (M-j+i-\sum_{t=i}^{j} l_t) \geq 0\ AND\ j \neq n && (1) \\ &= 0,\ if\ (M-j+i-\sum_{t=i}^{j} l_t) \geq 0\ AND\ j = n && (2) \\ &= \infty,\ if\ (M-j+i-\sum_{t=i}^{j} l_t) < 0 && (3) \end{aligned}$$

   Equation (2) captures the fact that the last line should not be charged a cost, if the words fit in one line.

Let us now define $m[i,j]$ to be the optimal cost of packing words w_i through w_j with word w_i starting on a fresh line. According to this definition, the solution to our problem is given by $m[1,n]$. The following recurrence is immediate:

$$\begin{aligned} m[i,j] &= s[i,j]\ if (M-j+i-\sum_{t=i}^{j} l_t) \geq 0 && (4) \\ &= \min_{i \leq k \leq j} s[i,k] + m[k+1,j],\ otherwise && (5) \end{aligned}$$