Quiz II – Solution

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1 Solution

Let us associate the decision variable x_i with job J_i, where

$$\begin{aligned} x_i &= 1, \; \textit{if } J_i \textit{ is processed by } A \\ &= 0, \; \textit{otherwise.} \end{aligned}$$

The state of the system is characterized by the decisions on the current variables and the current times spent on machine A and machine B.

We define $m[i, j, k]$ to be the minimum time taken to distribute jobs $J_1, J_2, \ldots, J_i$ assuming that we have at most time j to expend on machine A and at most time k to expend on machine B. Using this notation, we are interested in the value of $m[n, M, W]$, where

$$M = \sum_{i=1}^{n} a_i$$

$$W = \sum_{i=1}^{n} b_i$$

We further define $c[i, j, k]$ to be the time taken on machine A to realize $m[i, j, k]$ and likewise $d[i, j, k]$ to be the corresponding time taken on machine B. Clearly, $m[i, j, k] = \max\{c[i, j, k].d[i, j, k]\}$.

There are only two cases to consider for the decision on variable x_i.

1. $x_i = 1$- This means that J_i has been thrown to A. Hence, $c[i, j, k] = c[i, j - a_i, k] + a_i$ and $d[i, j, k] = d[i, j - a_i, k]$. The corresponding cost is denoted by m_a; hence $m_a = \max\{c[i, j - a_i, k] + a_i, d[i, j - a_i, k]\}$.

2. $x_i = 0$- This means that J_i has been thrown to B. Hence, $c[i, j, k] = c[i, j, k - b_i]$ and $d[i, j, k] = d[i, j, k - b_i] + b_i$. The corresponding cost is denoted by m_b; hence $m_b = \max\{c[i, j, k - b_i], d[i, j, k - b_i] + b_i\}$.

Finally, we can formulate the recurrence for $m[i, j, k]$ as follows:

$$m[i, j, k] = \min\{m_a, m_b\}$$

Remark: 1.1 *At least one of the arrays m, c, d, x can be eliminated!*

Remark: 1.2 *Convert the discussion above into a proper algorithm, implementable on a computer. The initialization steps are crucial! (5 points)*