Randomized Algorithms - Midterm

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1 Instructions

1. The Midterm is worth 30 points.
2. Please attempt all questions. You will receive partial credit for attempting problems.
3. You need to turn in your work on or before Thursday, October 18 (in class).

2 Problems

1. Consider the experiment of tossing a coin, which has probability p of turning up HEADS and $1-p$ of turning up TAILS on any one toss. We wish to study the random variable that characterizes the number of tosses required to obtain n heads. Write down the probability mass function of this random variable. Justify your answer. Calculate the mean and standard deviation of this distribution. (6 points)
2. In class, we studied the problem of selecting the k^{th} smallest element of an array using the LAZY-SELECT() algorithm. Let us now study the following algorithm for the same problem.

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Function SELECT-ITH-SMALLEST(A, p, q, i)
1: {This function finds the i^th smallest element in A[p..q]; consequently it will be invoked through SELECT-ITH-
   SMALLEST(A, 1, n, k), on an n element array}
2: if (p = r) then
3:    return(A[p])
4: end if
5: r ← PARTITION(A, p, q)
6: k ← r − p + 1
7: if (i = k) then
8:    return(A[r])
9: else
10:   if (i < k) then
11:      SELECT-ITH-SMALLEST(A, p, r − 1, i)
12:   else
13:      SELECT-ITH-SMALLEST(A, r + 1, q, i − k)
14:   end if
15: end if
```

Algorithm 2.1: Selection Algorithm

The PARTITION() function is identical to the one that we used to analyze QUICK-SORT(). What is the worst-case complexity of the algorithm. Assume that PARTITION() is implemented using randomization i.e.

the pivot element is chosen uniformly, at random; show that the expected time taken by Algorithm (2.1) is $O(n)$ (8 points).

3. Argue that $\mathbf{RP} \cap \mathbf{coRP} \subseteq \mathbf{ZPP}$ (4 points)
4. Exercise 3.6, Page 51 [MR95] (6 points)
5. Problem 3.1, Page 64, [MR95] (6 points)

References

[MR95] Rajeev Motwani and Prabhakar Raghavan. *Randomized Algorithms*. Cambridge University Press, Cambridge, England, June 1995.