Analysis of Algorithms -Midterm

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1 Instructions

1. This midterm is worth 30 points.
2. Induction always works!
3. Attempt as many problems as you can. You will be given partial credit.
4. The solutions will be posted on the Class Webpage, on Monday, October 7.
5. Your graded midterms will be returned on Friday, October 11.

2 Problems

1. Consider the recurrence relation (6 points):

$$\begin{aligned} T(1) &= 1 \\ T(n) &= 2 \cdot T(n-1) + 1, \quad n > 1 \end{aligned}$$

   Show that $T(n) = 2^n - 1$

2. Show that if $f(n) = O(g(n))$ and $e(n) = O(h(n))$, then $f(n) \cdot e(n) = O(g(n) \cdot h(n))$. (4 points)
3. Let $\mathbf{T}$ be a proper binary tree of height h, having n nodes. Show that $h \geq \log_2(n+1) - 1$. (6 points)
4. Consider the binary tree $\mathbf{T}$ in Figure (1). Write down the orders of the nodes, when you traverse the tree in inorder, preorder and postorder. (6 points)
5. Prove that Algorithm (2.1) correctly sorts an $n-$input sequence S provided as an $n-$element array $\mathbf{A}$ (in increasing order). You may assume that the n elements of the array are distinct and stored in the locations $A[1], A[2], \ldots, A[n]$. What is the worst-case running time of the algorithm? (8 points)

   Hint: You may either use the Loop Invariant Technique or induction (second principle!) on the number of elements in the array!

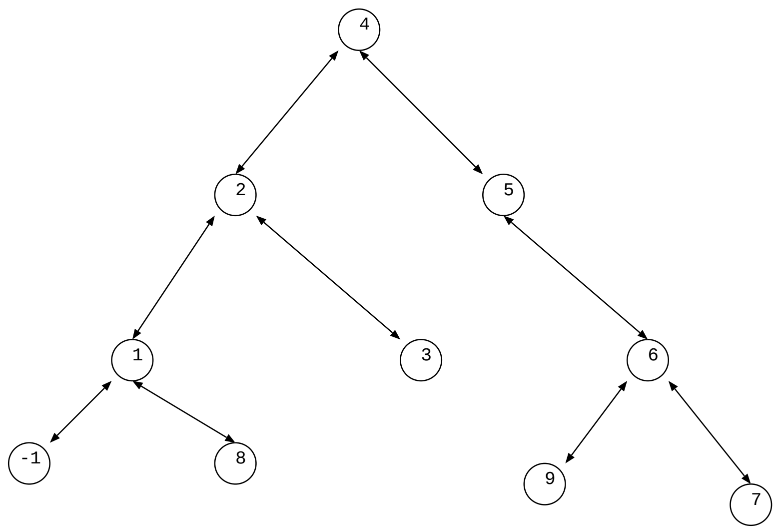

Figure 1: Binary Tree **T**

Function BUBBLE-SORT($\mathbf{A}, n$)

for $(i = 1$ **to** $n - 1)$ **do**
    for $(j = i + 1$ **to** $n)$ **do**
        if ($A[i] > A[j]$) **then**
            $temp = A[i]$
            $A[i] = A[j]$
            $A[j] = temp$
        end if
    end for
end for

Algorithm 2.1: Bubble Sort Algorithm