Analysis of Algorithms - Quiz I

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1 Instructions

1. There are 5 questions, each worth 2 points.
2. All logarithms are to base 2.
3. Induction always works!
4. Attempt as many problems as you can. You will be given partial credit.
5. The solutions will be posted on the Class Webpage, on Monday, September 23.

2 Problems

1. Prove using induction:

$$\sum_{i=1}^{n} i^2 = \frac{n \cdot (n+1) \cdot (2n+1)}{6}$$

2. Consider the recurrence relation:

$$\begin{aligned} T(n) &= 1, \ if \ \ n = 1 \\ &= T(n-1) + 2^n, \ \ otherwise \end{aligned}$$

   Show that $T(n) = 2^{n+1} - 3$.

3. Show that $\sum_{i=1}^{n} \log i = O(n \log n)$
4. Show that $\sum_{i=1}^{n} \log i = \Omega(n \log n)$
5. Let T be a proper binary tree of height h having n nodes. What is the minimum value for n as a function of h? Justify your answer.