Analysis of Algorithms - Quiz II

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1 Instructions

1. All logarithms are to base 2.
2. Induction always works!
3. Attempt as many problems as you can. You will be given partial credit.
4. The solutions will be posted on the Class webpage, on Monday, November 11.

2 Problems

1. Sort the sequence $\mathcal{S} = \{(3,3),(1,5),(2,5),(1,2),(2,3),(1,7),(3,2),(2,2)\}$, in increasing lexicographic order, using Radix sort, showing all the intermediate steps. Note that in lexicographic ordering $(x_1, y_1) < (x_2, y_2)$, *if* $x_1 < x_2$, *or* $x_1 = x_2$ *and* $y_1 < y_2$

   Radix sort is a Stable Sort procedure (2 points).

2. The coin changing problem is concerned with making change for n cents (n integral) using the fewest number of coins, where the coins are *quarters (*25 *cents), dimes (*10 *cents), nickels (*5 *cents)* and *pennies (*1 *cent).* For instance, we can make change for 10 cents using either 10 pennies or 2 nickels or 1 dime; clearly using 1 dime is the optimal solution, since it uses the fewest number of coins. Describe a greedy strategy for the coin changing problem. Argue that your strategy always derives the optimal solution for arbitrary n, i.e., it changes n into the *fewest number of coins*? (5 points)

3. Characterize the following recurrence (3 points):

$$\begin{aligned} T(n) &= 1, \; if \; n = 1 \\ &= 2 \cdot T(\frac{n}{2}) + \log n, \; n > 1 \end{aligned}$$

   Hint: Use Master's Theorem