Analysis of Algorithms - Scrimmage I

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Please attempt as many problems as you can in class. The scrimmage will not be graded, i.e. there are no points. The solutions are posted at:
`http://www.csee.wvu.edu/~ksmani/courses/fa02/cs320/cs320.html`

1. Prove using mathematical induction:
   (a) $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{n\cdot(n+1)} = \frac{n}{n+1}$.
   (b) $7^n - 2^n$ is divisible by 5.
   (c) Show that $13^n - 6^n$ is divisible by 7.
   (d) Show that

$$(cos\theta + i\ \sin\theta)^n = \ cos\ n\theta + i\ \sin\ n\theta\ \ \forall n \geq 1$$

   You may need to use the following identities from trigonometry

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$i^2 = -1$$

2. Compare $f(n)$ and $g(n)$ using asymptotic notation; you may either describe $f(n)$ in terms of $g(n)$ (for instance, $f(n) = O(g(n))$) or $g(n)$ in terms of $f(n)$ (for instance, $g(n) = \omega(f(n))$). Make sure that your description is as precise as possible.
   (a) $f(n) = n\log^5 n$, $g(n) = n^2$
   (b) $f(n) = n\log_4 n$, $g(n) = n\log_{35} n$
   (c) $f(n) = \log^3 n$, $g(n) = n^{\frac{1}{3}}$
   (d) $f(n) = 2^n$, $g(n) = 2^{n+1}$