Advanced Analysis of Algorithms - Midterm

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1 Instructions

1. Attempt as many problems as you can. You will be given partial credit.
2. The duration of this quiz is 1 hour, 25 minutes, i.e., 8 : 00-9 : 25 am.
3. Each question is worth 6 points for a total of 30 points.

2 Problems

1. `Summation:` Obtain asymptotic tight bounds (upper and lower) on

$$\sum_{k=1}^{n} k^2 \log k.$$

2. `Convexity:`
   (a) Let S_1 and S_2 be 2 convex sets. Argue that $S_1 \cap S_2$ is a convex set. (3 points.)
   (b) What can you say about the function $f(x) = \sin x$ in the interval $[0, \pi]$, as regards convexity? Justify your answer mathematically. (3 points)
3. `Probability:` Consider a bin containing 5 red balls and 7 black balls. What is the probability of obtaining 2 red balls in a single draw of 2 balls, where a draw of 2 balls is defined as:
   (a) One ball is drawn from the bin and then the second one is drawn, without replacing the first ball. (2 points)
   (b) One ball is drawn from the bin and the second one is drawn, after replacing the ball drawn first in the bin. (2 points)
   (c) The two balls are *selected* at once from the bin. (2 points)
4. `Search Trees:` Argue that if a node in a binary search tree has 2 children, then its successor has no left child, while its predecessor has no right child.
5. `Dynamic Programming:` Let $S = \{s_1, s_2, \ldots, s_n\}$ denote a collection of n positive numbers, such that $\sum_{i=1}^{n} s_i = N$. Devise an algorithm that runs in time $O(n \cdot N)$ to check if there is a set $S' \subseteq S$, such that $\sum_{s_i \in S'} s_i = \sum_{s_i \in S - S'} s_i$. (*Hint:* 0/1 *Knapsack!*)