Advanced Analysis of Algorithms - Practice Midterm

L. Kovalchick
LCSEE,
West Virginia University,
Morgantown, WV
{lynn@csee.wvu.edu}

Please attempt as many problems as you can in class. The practice midterm will not be graded, i.e. there are no points. The solutions are posted at:
`http://www.csee.wvu.edu/~ksmani/courses/fa03/algos/algos.html`

1 Problems

1. Show that

$$\sum_{i=1}^{n} i^3 = [\frac{n \cdot (n+1)}{2}]^2.$$

2. Consider the recurrence relation:

$$\begin{aligned} T(n) &= 1, \ if \ \ n = 1 \\ &= T(n-1) + 2^n, \ \ otherwise \end{aligned}$$

   Show that $T(n) = 2^{n+1} - 3$.

3. Give an algorithm which will return the third largest element of a heap with $heap - size \geq 3$, in $O(1)$ time. Assume that all elements are unique.

4. Describe an algorithm that, given n integers in the range from 1 to k, preprocesses its input and then answers any query about how many of the n integers fall into a range from $[a \cdots b]$ in $O(1)$ time. Your algorithm should use $O(n + k)$ preprocessing time.

5. Suppose a biker is about to go on a ride on a bike trail carrying a single knapsack. Suppose further that she knows the maximum total weight W that she can carry, and she has a set S of n different useful items that she can potentially take with her. Let us assume that each item j has an integer weight w_j and a benefit value b_j, which is a value that the biker assigns to item j. Her problem is to optimize the value of the items that she places into her knapsack, without going over the weight limit W (i.e., maximize $\sum_{j \in T} b_j$ subject to $\sum_{j \in T} w_j \leq W$). Give a recursive definition for the profit value of the optimal solution to this problem.