Automata Theory - Final

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1 Instructions

- (i) The final should be returned by 5:30 pm on 12/13/04.
- (ii) Each question is worth 5 points.
- (iii) Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.
- (iv) Feel free to quote any Theorem which was either proved in class or occurs in the textbook.
- (v) The solutions have been posted on the class URL.

2 Problems

- 1. Let $\Sigma = \{a, b\}$. Write a CFG for the language $L = \{w \mid w = a^i b^{2i}, i > 0\}.$
- 2. (a) Argue that the following CFG is ambiguous. (2 points)

$$\begin{array}{rcl}
E & \rightarrow & E - E \\
E & \rightarrow & 0 \mid 1
\end{array} \tag{1}$$

- (b) Write an unambiguous CFG for the language represented by the CFG of System (1). (3 points)
- 3. Let $\Sigma = \{0, 1\}$. A string $w \in \Sigma^*$ is said to be *balanced*, if it contains an equal number of 0's and 1's. Consider the CFG represented by System (2)

$$\begin{array}{rcl} S & \to & \epsilon \\ S & \to & SS \\ S & \to & 0S1 \mid 1S0 \end{array} \tag{2}$$

Argue that this CFG represents the language of *all* and *only* balanced strings over the alphabet Σ . *Hint: Recall the Prefix Theorem proved in class.*

- In class, we proved that if Σ is a finite alphabet, then Σ* is a countable set. What can you say about the set 2^{Σ*}, as regards countability?
- 5. Let $\Sigma = \{(,)\}$. A string $w \in \Sigma^*$ is said to be *parenthetically balanced*, if the following two conditions are met:
 - (a) The total number of left parentheses and right parentheses in w are equal, and
 - (b) In each prefix of w, the number of left parentheses is at least as large as the number of right parentheses.

For instance, "(((())))" is parenthetically balanced, whereas ")(" and "(()" are not. Design a Pushdown Automaton that accepts the language of parenthetically balanced strings over the alphabet Σ .

Design a Deterministic Turing Machine that *decides* the language L = {0ⁿ · 1ⁿ, n ≥ 0}.

3 Important Definitions

Definition 3.1 Prefix - Let $w = a_1a_2...a_n$ denote a string over an alphabet Σ . Any substring $a_1a_2...a_i$, $i \leq n$ is called a prefix of w. It is important that the prefix string and the original string match identically in their character positions, for the length of the prefix string.

Definition 3.2 Power Set - Given a set A, the set of all subsets of A, is called the Power Set of A and is denoted by 2^A . For instance, if $A = \{0,1\}$, then $2^A = \{\phi, \{0\}, \{1\}, \{0,1\}\}$.