Automata Theory - Homework I

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1 Instructions

1. The homework is due on September 14, in class.
2. Each question is worth 2 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Let A and B be propositions. Argue that the following two statements are tautologies:
   (a) $A \rightarrow A$,
   (b) $[A \wedge (A \rightarrow B)] \rightarrow B$
2. Explain the difference between the *converse* of a theorem and its *contra-positive*.
3. Use mathematical induction to show that $7^n - 2^n$ is divisible by 5, for all $n \geq 0$.
4. Let S and T denote two sets, which are subsets of a set U. Let S' and T' denote the complements of S and T in U respectively. Prove the following set equivalence:
$$(S \cup T)' = S' \cap T'.$$
5. Let $\Sigma = \{0, 1\}$ denote an alphabet. Enumerate five elements of the following languages:
   (a) Even binary numbers,
   (b) The number of zeros is not equal to the number of ones in a binary string.