## Automata Theory - Midterm

K. Subramani LCSEE, West Virginia University, Morgantown, WV {ksmani@csee.wvu.edu}

## **1** Instructions

- 1. The midterm should be returned by 9:15 am on 10/7/04.
- 2. Each question is worth 5 points.
- Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.
- 4. Feel free to quote any Theorem from the book; however, any "law" that you use, must be proved.

## 2 Problems

- 1. Professor Chikovski wants to prove the conjecture, "If *B* then *C*". After working for four hours, he succeeds in proving the theorem, "If *A* then *B* and *C*". His graduate student points out to him that the theorem, "If *B* then *A*" is a well known fact. Can the Professor now claim that his conjecture, "If *B* then *C*" is a theorem? If so, provide a proof of the same. If not, provide a counterexample.
- 2. Formally prove that the DFA described by the transition table below, accepts all and only those binary strings which do not contain two consecutive 0's.

	0	1
$\rightarrow * q_0$	$q_1$	$q_0$
$* q_1$	$q_2$	$q_0$
$q_2$	$q_2$	$q_2$

- Suppose that you are given a DFA A = (Q, Σ, δ, q<sub>0</sub>, F), which accepts the language L ⊆ Σ\*. Let us say that we wish to design a DFA that accepts the language L<sup>c</sup>, where L<sup>c</sup> = {w | w ∈ Σ\* and w ∉ L}.
  - (i) Argue using induction that the DFA
     A<sup>c</sup> = (Q, Σ, δ, q<sub>0</sub>, Q - F) serves the purpose.
     (2 points)

- (ii) Will the same trick work if A is an NFA. If so, provide a formal proof of the same. If not, provide a counterexample. (3 points)
- 4. (a) Convert the regular expression 01\* to a DFA. (2 points)
  - (b) Write a regular expression for the DFA described by the following transition table: (3 points)

	0	1
$\rightarrow q_1$	$q_2$	$q_1$
$q_2$	$q_3$	$q_1$
$* q_3$	$q_3$	$q_2$

- 5. Prove or disprove the following laws on regular expressions:
  - (i)  $(R+S)^*S = (R^*S^*)^*$ . (2 points)
  - (ii) R(S+T) = RS + RT. (3 points)
- 6. Let  $\Sigma = \{0, 1\}$ . Argue that the language  $L = \{0^i \cdot 1^j \mid i \ge 0, i \le j\}$  is not regular.