Analysis of Algorithms - Homework I

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1 Instructions

1. The homework is due on September 14, in class.
2. Each question is worth 3 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Write a recursive algorithm to compute the maximum element in an array of integers. You may assume the existence of a function "$\max(a,\ b)$" that returns the maximum of *two* integers a and b.
2. Argue that your algorithm is correct. *Hint: First Principle of Mathematical Induction.*
3. What is the *exact* comparison complexity of your algorithm? Derive a recurrence relation and solve it to justify your answer.
4. Argue using induction that the exact solution to the recurrence relation:

$$\begin{aligned} T(1) &= 0 \\ T(n) &= 2 \cdot T(\frac{n}{2}) + n,\ n \geq 2 \end{aligned}$$

   is $T(n) = n \cdot \log n$.
5. Show that $log(n!) \in O(n \cdot \log n)$.