Analysis of Algorithms - Homework II

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1 Instructions

1. The homework is due on October 28, in class.
2. Each question is worth 3 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Professor Stankowski proposes the following algorithm for sorting an array $\mathbf{A}$ of n numbers:

   (i) If there is only one number, return.

   (ii) If there are two numbers, perform a single comparison to determine the order.

   (iii) If there are more than two numbers, then first sort the top two-thirds of the elements recursively. Follow this by sorting the bottom two-thirds of the elements recursively and then sorting the top two-thirds of the elements again.

   Write a recursive algorithm to implement the above strategy and argue the correctness of Professor Stankowski's algorithm.

2. What is the comparison complexity of Professor Stankowski's algorithm? Formulate a recurrence relation and solve the same to justify your answer.

3. Describe how you would implement a queue data structure using two stacks. In particular, describe algorithms for the INSERT() and DELETE() operations, assuming that PUSH() and POP() functions have been implemented.

4. The path length of a binary tree is defined as the sum of the depths of all the nodes in the tree. Write a linear time algorithm to compute the path length of an arbitrary binary tree.

5. In the single machine scheduling problem, you are given a single machine and a collection of tasks $T = \{T_1, T_2, \ldots, T_n\}$ with respective start times $S = \{s_1, s_2, \ldots, s_n\}$ and respective finish times $F = \{f_1, f_2, \ldots, f_n\}$. In other words task T_i starts at time s_i and finishes at time f_i. The machine can execute precisely one task at a time; accordingly, tasks scheduled on the machine must be non-conflicting. The goal is to maximize the number of tasks scheduled on the machine. Design a greedy algorithm for this problem and argue its correctness.