Analysis of Algorithms - Midterm

K. Subramani
LCSEE,
West Virginia University,
Morgantown, WV
{ksmani@csee.wvu.edu}

1 Instructions

1. The Midterm is to be turned in by $9:00$ *am.* in class.
2. Each question is worth 4 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. **Asymptotics:**
   (a) Show that $[(f(n) \in \Omega(g(n))) \land (g(n) \in \Omega(h(n)))] \Rightarrow f(n) \in \Omega(h(n))$.
   (b) Does $\log^3 n \in O(n^{0.5})$?
2. **Algorithm Design for order:** Given an integer array of n elements, design an algorithm to find both the maximum element and the minimum element, using at most $\frac{3n}{2}$ element to element comparisons. Comparisions for iterators (e.g., **for** loops) do not count.
3. **Binary Search Trees:** Enumerate all the binary search trees on the keys 1, 2 and 3.
4. **Sorting:** Explain briefly how Randomized Quicksort performs $O(n \cdot \log n)$ comparisons, in the expected case, to sort an array of n elements. (You may assume the algorithm discussed in class.)
5. **Properties of Binary Trees:** Let T be a proper binary tree with n nodes and height h. Argue that the number of external nodes in T is at least $h+1$ and at most 2^h.