Analysis of Algorithms - Quiz I

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1 Instructions

1. The Quiz is to be returned by $9:00$ *am.* in class.
2. Each question is worth 3 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Show that $\log(n!) \in \Omega(n \cdot \log n)$.
2. Let T be a proper binary tree with height h having n nodes. Show that $\log_2(n+1) - 1 \leq h$, i.e., the height of any proper binary tree having n nodes is at least $\log_2(n+1) - 1$.
3. Argue using mathematical induction that the solution to the recurrence

$$\begin{aligned} T(n) &= 1, \text{ if n} = 1 \\ &= T(n-1) + n, \text{ n} \geq 2 \end{aligned}$$

   is $T(n) = \frac{n \cdot (n+1)}{2}$.
4. Write a recursive algorithm for the *Post-order* Traversal of a binary tree. Argue using induction that all nodes in the tree are reached by your algorithm.
5. Assume that you are given a rudimentary programming language which contains only four operators, viz., $+$, $-$, abs and div. $+$ and $-$ have their usual meanings, while $div(a,b)$ returns the quotient of $\frac{a}{b}$ and $abs(a)$ returns the absolute value of a. Write a function $\max(a,b)$ that takes two integers a and b as input and returns the maximum of the two. Note that you can only use the operators provided; in particular, the constructs "**if**", "**while**", or "**for**" are not available.