1 Instructions

1. The Quiz is to be returned by 9:00 am in class.
2. Each question is worth 3 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Recurrences: Solve the following recurrences using the Master method:

   (i) \[ T(1) = 0 \]
   \[ T(n) = 2 \cdot T\left(\frac{n}{2}\right) + \log n, \quad n > 1 \]

   (ii) \[ T(1) = 0 \]
   \[ T(n) = 9 \cdot T\left(\frac{n}{3}\right) + n^3 \log n, \quad n > 1 \]

2. Divide-And-Conquer (Application) Use Strassen’s matrix multiplication algorithm to multiply
   \[ X = \begin{bmatrix} 3 & 2 \\ 4 & 8 \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} 1 & 5 \\ 9 & 6 \end{bmatrix}. \]

3. Divide-And-Conquer (Theory) Design a Divide-And-Conquer strategy to find both the maximum and the minimum elements of an integer array using at most \(\frac{3n}{2}\) comparisons. Analyze your algorithm through a recurrence relation. Note that the strategy discussed in the Midterm solutions is not Divide-And-Conquer.

4. Greedy: Let \(G = \langle V, E \rangle\) denote an undirected graph with vertex set \(V\) and edge set \(E\). Assume that the weights on the edges of \(G\) are distinct, i.e., no two edges have the same weight. Argue that \(G\) has a unique Minimum Spanning Tree. Hint: Recall the proof of correctness of Kruskal’s algorithm and modify it ever so slightly!

5. Dynamic Programming: Assume that you are given a chain of matrices \(< A_1 A_2 A_3 A_4 >\), with dimensions \(2 \times 5, 5 \times 4, 4 \times 2\) and \(2 \times 4\) respectively. Compute the optimal number of multiplications required to calculate the chain product.