Analysis of Algorithms - Quiz II

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1 Instructions

1. The Quiz is to be returned by $9:00$ *am.* in class.
2. Each question is worth 3 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. **Recurrences:** Solve the following recurrences using the Master method:

   (i)

   $$\begin{aligned} T(1) &= 0 \\ T(n) &= 2 \cdot T(\frac{n}{2}) + \log n, \; n > 1 \end{aligned}$$

   (ii)

   $$\begin{aligned} T(1) &= 0 \\ T(n) &= 9 \cdot T(\frac{n}{3}) + n^3 \log n, \; n > 1 \end{aligned}$$

2. **Divide-And-Conquer (Application)** Use Strassen's matrix mutiplication algorithm to multiply

   $$\mathbf{X} = \begin{bmatrix} 3 & 2 \\ 4 & 8 \end{bmatrix} \text{ and } \mathbf{Y} = \begin{bmatrix} 1 & 5 \\ 9 & 6 \end{bmatrix}.$$

3. **Divide-And-Conquer (Theory)** Design a Divide-And-Conquer strategy to find both the maximum and the minimum elements of an integer array using at most $\frac{3n}{2}$ comparisons. Analyze your algorithm through a recurrence relation. Note that the strategy discussed in the Midterm solutions is *not* Divide-And-Conquer.

4. **Greedy:** Let $G =< \mathbf{V}, \mathbf{E} >$ denote an undirected graph with vertex set $\mathbf{V}$ and edge set $\mathbf{E}$. Assume that the weights on the edges of G are distinct, i.e., no two edges have the same weight. Argue that G has a unique Minimum Spanning Tree. *Hint: Recall the proof of correctness of Kruskal's algorithm and modify it ever so slightly!*

5. **Dynamic Programming:** Assume that you are given a chain of matrices $< A_1 \; A_2 \; A_3 \; A_4 >$, with dimensions 2×5, 5×4, 4×2 and 2×4 respectively. Compute the optimal number of multiplications required to calculate the chain product.