Analysis of Algorithms - Scrimmage I

K. Subramani
LCSEE,
West Virginia University,
Morgantown, WV
{ksmani@csee.wvu.edu}

1 Instructions

1. The scrimmage will not be graded.
2. Attempt as many problems as you can.
3. The solutions have been posted on the class URL.

2 Problems

1. Prove that
$$\sum_{i=1}^{n} i^2 = \frac{n \cdot (n+1) \cdot (2n+1)}{6}.$$
2. Given $a > 0$ and $0 < r < 1$, argue that
$$\sum_{i=0}^{n} a \cdot r^i = \frac{a \cdot (1 - r^{n+1})}{1 - r}$$
3. Let T denote a proper binary tree. Show that the maximum number of nodes in level i is 2^i.
4. Let T denote a proper binary tree with n nodes and height h. Argue that $h \le \frac{n-1}{2}$.
5. Argue using induction that the exact solution to the recurrence relation:
$$\begin{aligned} T(0) &= 1 \\ T(n) &= 2 \cdot T(n-1), \; n \ge 1 \end{aligned}$$
is $G(n) = 2^n$.
6. Argue that $2^n \in \Omega(5^{\log n})$