1 Instructions

1. The scrimmage will not be graded.
2. Attempt as many problems as you can.
3. The solutions have been posted on the class URL.

2 Problems

1. Prove that

\[ \sum_{i=1}^{n} i^2 = \frac{n \cdot (n + 1) \cdot (2n + 1)}{6}. \]

2. Given \( a > 0 \) and \( 0 < r < 1 \), argue that

\[ \sum_{i=0}^{n} a \cdot r^i = \frac{a \cdot (1 - r^{n+1})}{1 - r}. \]

3. Let \( T \) denote a proper binary tree. Show that the maximum number of nodes in level \( i \) is \( 2^i \).

4. Let \( T \) denote a proper binary tree with \( n \) nodes and height \( h \). Argue that \( h \leq \frac{n-1}{2} \).

5. Argue using induction that the exact solution to the recurrence relation:

\[
\begin{align*}
T(0) &= 1 \\
T(n) &= 2 \cdot T(n - 1), \quad n \geq 1
\end{align*}
\]

is \( G(n) = 2^n \).

6. Argue that \( 2^n \in \Omega(5^{\log n}) \)