Analysis of Algorithms - Scrimmage II

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1 Instructions

1. The scrimmage will not be graded.

2. Attempt as many problems as you can.

3. The solutions have been posted on the class URL.

2 Important Concepts

Definition 2.1 *$O(g(n))$ is the class of functions that do not grow faster than $g(n)$, as n tends to ∞. Mathematically, $f(n) \in O(g(n))$, if $f(n) \leq c_1 \cdot g_n$, for some $c_1 > 0$ and all $n \geq N_0$, where N_0 is some fixed constant.*

Definition 2.2 *$\Omega(g(n))$ is the class of functions that grow at least as fast as $g(n)$, as n tends to ∞. Mathematically, $f(n) \in \Omega(g(n))$, if $f(n) \geq c_1 \cdot g_n$, for some $c_1 > 0$ and all $n \geq N_0$, where N_0 is some fixed constant.*

Definition 2.3 *$\Theta(g(n))$ is the class of functions that grow* exactly *as fast as $g(n)$. Mathematically, $f(n) \in \Theta(g(n))$, if* $f(n) \in O(g(n))$ and $f(n) \in \Omega(g(n))$.

3 Techniques

The four common techniques for establising asymptotic orders are:

(i) Induction - Works only when the answer is known exactly; can also be applied when an approximate answer is known, but is usually messy.

(ii) Summation Rules - See “Tools of the Trade” handout.

(iii) Limit Rules - If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = k_1$, then,

  (a) $k_1 = 0 \Rightarrow f(n) \in O(g(n))$.

  (b) $k_1 = \infty \Rightarrow f(n) \in \Omega(g(n))$.

  (c) $k_1\ is \ \text{some constant, not zero} \Rightarrow f(n) \in \Theta(g(n))$.

  Limit rules may not always apply (See Quiz I).

(iv) Clever Observations - See Solutions to Quiz I and Homework I.

4 Problems

1. Show that if $f(n)$ and $g(n)$ are monotonically increasing functions, then so is $f(n) + g(n)$.
2. Prove that $n! \in \Omega(2^n)$.
3. Prove that $f(n) + O(f(n)) \in O(f(n))$.
4. Prove that

$$1^2 + 3^2 + \ldots (2n-1)^2 = \frac{n \cdot (2n-1) \cdot (2n+1)}{3}.$$