Analysis of Algorithms - Tools of the trade

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1 Important Techniques

(a) Summation - Used to count number of steps in loops and recursive calls. Also used in counting number of nodes in a binary tree.

(b) Induction - Used in solving recurrences, summations and proving correctness of algorithms.

2 Important Identities

(a) Sum of a Geometric Progression - For $a > 0$, $r > 0$

$$\begin{aligned} \sum_{i=0}^{n} a \cdot r^i &= \frac{a \cdot (1 - r^{n+1})}{1 - r}, \text{ if r} < 1 \\ &= \frac{a \cdot (r^{n+1} - 1)}{r - 1}, \text{ if r} > 1 \end{aligned}$$

(b) L'Hospital's rule - If $\lim_{n \to \infty} f(n) \to \infty$ and $\lim_{n \to \infty} g(n) \to \infty$, then $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{f'(n)}{g'(n)}$, where $f'(n)$ denotes the first derivative of $f(n)$.

(c) $\int \log x = x \log x - x$.

(d) Logarithm rules:

(i) $\log a + \log b = \log ab$.

(ii) $\log a - \log b = \log \frac{a}{b}$.

(iii) $\log a^c = c \cdot \log a$.

(iv) $b^{\log_a n} = n^{\log_a b}$.

(e) Bounding sums by integrals - A function $f(x)$, is said to be non-decreasing, if $a \leq b \Rightarrow f(a) \leq f(b)$. For example, x^2 and $\log x$ are non-decreasing functions. Likewise, a function $f(x)$, is said to be non-increasing, if $a \leq b \Rightarrow f(a) \geq f(b)$. For example, $\frac{1}{x}$ and $-x$ are non-increasing functions.

If $f(i)$ is an non-decreasing function,

$$\int_{a-1}^{b} f(x) \cdot dx \leq \sum_{i=a}^{b} f(i) \leq \int_{a}^{b+1} f(x) \cdot dx$$

If $f(i)$ is a non-increasing function,

$$\int_{a-1}^{b} f(x) \cdot dx \geq \sum_{i=a}^{b} f(i) \geq \int_{a}^{b+1} f(x) \cdot dx$$