Advanced Analysis of Algorithms - Homework I

K. Subramani
LCSEE,
West Virginia University,
Morgantown, WV
{ksmani@csee.wvu.edu}

1 Instructions

1. The homework is due on September 15, in class. Each question is worth 4 points.
2. You may assume that all function placeholders are monotonically non-decreasing.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Given an array $\mathbf{A}$ of n integer elements, how would you find the second smallest element in $n + \log_2 n$ comparisons.
2. Indicate whether each of the following identities is true or false, giving a proof if true and a counterexample otherwise.
   (a) $f(n) + o(f(n)) \in \Theta(f(n))$.
   (b) $(f(n) \in O(g(n))) \wedge (g(n) \in O(h(n))) \Rightarrow (f(n) \in O(h(n)))$.
   (c) $\log^{1/\epsilon} n \in O(n^{\epsilon})$, $(\forall \epsilon)\ 0 < \epsilon < 1$.
   (d) $2^n \in \Omega(5^{\log_e n})$.
3. Devise a Divide-and-Conquer procedure for computing the k^{th} largest element in an array of integers. Analyze the asymptotic time complexity of your algorithm. (*Hint: Use the Partition procedure discussed in class.*)
4. Argue the correctness of the MERGE() procedure discussed in class. (*Hint: Write a recursive version of* MERGE() *and then use induction.*)
5. What is the value returned by Algorithm (2.1) when called with $n = 10$?

```
Function LOOP-COUNTER(n)
1: count = 0
2: for (i = 1 to n) do
3:    for (j = 1 to i) do
4:       for (k = 1 to j) do
5:          count++
6:       end for
7:    end for
8: end for
9: return(count)
```

Algorithm 2.1: Loop Counter