Advanced Analysis of Algorithms - Homework II

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1 Instructions

1. The homework is due on October 6, in class. Each question is worth 4 points.
2. The only material that you are permitted to refer is the course textbook.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Assume that you are given the following weight matrix for a four vertex graph. Compute the shortest-path distances between each pair of vertices in the graph using the Floyd algorithm. You are required to present all the intermediate distance matrices.

$$\mathbf{W} = \begin{bmatrix} 0 & 5 & \infty & 3 \\ 4 & 0 & 4 & \infty \\ 2 & 1 & 0 & \infty \\ 3 & 2 & \infty & 0 \end{bmatrix}$$

2. Assume that you are given a graph $\mathbf{G}$ with some edges having positive weights and some edges having negative weights. How would you use the Floyd algorithm for checking whether $\mathbf{G}$ has a negative cost cycle?
3. Let $\mathbf{X} =< x_1, x_2, \ldots x_m >$ and $\mathbf{Y} =< y_1, y_2, \ldots, y_n >$ denote two sequences on a fixed alphabet Σ (You may assume that $\Sigma = \{0, 1\}$, if it helps). Devise an efficient algorithm to determine the length of the Longest Common Subsequence between $\mathbf{X}$ and $\mathbf{Y}$. Note that a subsequence of a sequence need not be contiguous.
Hint: Use Dynamic Programming and derive a recurrence relation.
4. A Directed Acyclic Graph (DAG) is a graph without any cycles. The Longest Path problem on a graph is concerned with computing the length of the longest simple path between each pair of vertices in that graph. Given a DAG $\mathbf{D}$, argue that the Longest Path problem does satisfy the optimal substructure property demanded by dynamic programming. Derive a recurrence relation for this problem.
5. Assume that you are given a chain of matrices $< A_1\ A_2\ A_3\ A_4 >$, with dimensions 2×5, 5×4, 4×2 and 2×4 respectively. Compute the optimal number of multiplications required to calculate the chain product and also indicate what the optimal order of multiplication should be using parentheses.