

# Advanced Analysis of Algorithms - Homework III

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## 1 Instructions

1. The homework is due on November 17, in class. Each question is worth 4 points.
2. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

## 2 Problems

1. Is there a problem in the complexity class  $\mathcal{P}$ , such that all problems in  $\mathcal{P}$  can be polynomially transformed to this problem?
2. Show that a language  $L$  can be *verified* in deterministic polynomial time if and only if it can be decided by a non-deterministic algorithm in polynomial time.
3. Design a backtracking algorithm for the 3SAT problem.
4. Consider an instance of the Subset-Sum problem, where  $S = \{2, 10, 13, 17, 22, 42\}$  and  $B = 52$ . Solve this instance using backtracking, showing all the steps.
5. Consider the following graph coloring algorithm for coloring the vertices of a graph using the fewest number of colors:

**Function** FIND-OPTIMAL-COLOR( $\mathbf{G}=\langle \mathbf{V}, \mathbf{E} \rangle$ )

- 1: Let  $V_{un} = V$  and  $C_u = \{1, 2, \dots, n\}$ .
- 2: **while** ( $V_{un} \neq \phi$ ) **do**
- 3:    $c_{cur}$  is the smallest indexed color in  $C$ .
- 4:   Assign  $c_{cur}$  to as many vertices as possible in  $V_{un}$  making sure that a vertex with index number  $k$  is considered before a vertex with index number  $k + 1$ .
- 5:   Delete all the colored vertices from  $V_{un}$ .
- 6:   Delete  $c_{cur}$  from  $C$ .
- 7: **end while**

**Algorithm 2.1:** Graph Coloring Algorithm

$V_{un}$  is the set of uncolored vertices and  $C_u$  is the set of unassigned colors.

Is Algorithm (2.1) optimal? Justify your answer with a proof or a counterexample.