Advanced Analysis of Algorithms - Midterm

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1 Instructions

1. The midterm is to be turned in by 9:15 am.
2. Each question is worth 4 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Solve the following recurrence using substitution:

$$\begin{aligned} T(1) &= 0 \\ T(n) &= 2 \cdot T(\frac{n}{2}) + n \cdot \log n,\ n \geq 2 \end{aligned}$$

2. Given an array $\mathbf{A}$ of n integer elements, design an algorithm that computes the number of inversion pairs and runs in $O(n \cdot \log n)$ time. Note that an inversion pair is a pair of indices (i, j), such that $i < j$ and $\mathbf{A}[i] > \mathbf{A}[j]$.
Hint: Use Divide-and-Conquer.

3. Construct the optimal binary search tree on the following four ordered keys, $key_1 \leq key_2 \leq key_3 \leq key_4$, with probability distribution $p_1 = \frac{1}{2}$, $p_2 = \frac{1}{8}$, $p_3 = \frac{1}{8}$ and $p_4 = \frac{1}{4}$.

4. In the Fractional Knapsack problem, you are given n objects $O = \{o_1, o_2, \ldots, o_n\}$ with respective weights $W = \{w_1, w_2, \ldots, w_n\}$ and respective profits $P = \{p_1, p_2, \ldots, p_n\}$. The goal is to pack these objects into a knapsack of capacity M, such that the profit of the objects in the knapsack is maximized, while the weight constraint is not violated. You may choose a fraction of an object, if you so decide; if $\alpha_i,\ 0 \leq \alpha_i \leq 1$ of object o_i is chosen, then the profit contribution of this object is $\alpha_i \cdot o_i$ and its weight contribution is $\alpha_i \cdot w_i$. Design a greedy algorithm for this problem and argue its correctness.

5. Argue that Randomized Quicksort takes $O(n \cdot \log n)$ comparisons, in the expected case, to sort an array of n elements.