Analysis of Algorithms - Homework II

K. Subramani
LCSEE,
West Virginia University,
Morgantown, WV
{ksmani@csee.wvu.edu}

1 Instructions

1. The homework is due on October 29, in class.
2. Each question is worth 3 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. In the Knapsack problem, you are given a knapsack of capacity W and n objects $\{o_1, o_2, \ldots o_n\}$ with respective weights $\{w_1, w_2, \ldots, w_n\}$ and respective profit values $\{p_1, p_2, \ldots p_n\}$. The goal is to pack the objects into the knapsack in a manner that maximizes the profit of knapsack, without violating its capacity constraint. In class, we showed that if we are permitted to choose fractions of objects, then the problem can be solved by a greedy strategy. The $0/1$ Knapsack problem is a variant of the knapsack problem in which you *cannot* choose fractions of objects, i.e., each object is either selected or not. Argue with a counterexample that the greedy strategy discussed in class does not work for the $0/1$ Knapsack problem.
2. Let $A = \{a_1, a_2, \ldots, a_n\}$ denote a set of positive integers that add up to N. Design an $O(n \cdot N)$ algorithm for determining whether there is a subset B of A, such that $\sum_{a_i \in B} a_i = \sum_{a_i \in A-B} a_i$.
3. Let G be an undirected spanning tree on n vertices and m edges. Argue that $m = n - 1$.
4. Consider the DFS algorithm discussed in class. Argue that for any pair of nodes u and v, precisely one of the following two possibilities holds:
   (i) $[d[u], f[u]] \cap [d[v], f[v]] = \emptyset$.
   (ii) Either $[d[u], f[u]]$ is completely contained in $[d[v], f[v]]$ or vice versa.
5. Consider a directed weighted graph in which all edges, except those directed out of the source are positive. The edges out of the source can be positive or negative. Will Dijkstra's algorithm produce the correct shortest path distances when run on this graph? Justify your answer with a proof or a counterexample.