Analysis of Algorithms - Midterm

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1 Instructions

1. The Midterm needs to be turned in by $8:50$ am on October 8.
2. Each question is worth 4 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. **Recurrences:** Solve the following recurrences exactly or asymototicaly. You may assume any convenient form for n.

   (a)

   $$\begin{aligned} T(1) &= 1 \\ T(n) &= T(\sqrt[3]{n}) + 1, \; n > 1 \end{aligned}$$

   (b)

   $$\begin{aligned} T(1) &= 0 \\ T(n) &= 4T(\frac{n}{2}) + n^2 \cdot \log n, \; n > 1 \end{aligned}$$

2. **Binary Trees:** Let T denote a proper binary tree with n internal nodes. We define $E(T)$ to be the sum of the depths of all the external nodes of T; likewise, $I(T)$ is defined to be the sum of the depths of all the internal nodes of T. Prove that $E(T) = I(T) + 2 \cdot n$.

3. **Greedy:** Assume that you are given a set S of n activities $\{a_1, a_2, \ldots, a_n\}$. Associated with activity a_i are its start time s_i and finish time f_i; if activity a_i is selected then it *must* start at s_i and finish before f_i. Two activities a_i and a_j are compatible, if $s_i \geq f_j$ or $s_j \geq f_i$. Design an algorithm that outputs the largest set of compatible activities.

4. **Sorting:** Analogous to the notion of worst-case running time for an algorithm, is the notion of *best-case* running time, which is the minimum amount of time that an algorithm needs to accomplish its task. Argue that the best-case running time of Quicksort (in terms of element-to-element comparisons) is $\Omega(n \cdot \log n)$. (It is interesting to note that the best-case running time of Insertion sort is $O(n)$.)

5. **Divide and Conquer:** Design a *Divide-And-Conquer* algorithm to discover both the maximum and minimum of an array $\mathbf{A}$ of n elements using at most $\frac{3n}{2}$ element-to-element comparisons. Formally prove that your algorithm makes at most $\frac{3}{2}n$ element-to-element comparisons.