Analysis of Algorithms - Quiz I

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1 Instructions

1. The quiz needs to be turned in by $8:50$ am.
2. Each question is worth 3 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. **Recurrences:**
   (a) Solve the following recurrence. You may assume any convenient form for n.
   $$\begin{aligned} T(1) &= 0 \\ T(n) &= T(\frac{n}{2}) + 1, \ n > 1 \end{aligned}$$
   (b) Consider the following recurrence relation:
   $$\begin{aligned} T(1) &= 4 \\ T(n) &= T(n-1) + 4 \end{aligned}$$
   Argue using mathematical induction that $T(n) = 4 \cdot n$. Note that you *must* use induction to establish the solution.
2. **Asymptotics:**
   (a) Show that if $f(n) \in O(g(n))$ and $g(n) \in O(h(n))$, then $f(n) \in O(h(n))$.
   (b) Show that for any function $f(n)$, the set $\omega(f(n)) \cap o(f(n))$ is empty.
3. **Greedy:** In class, we established the correctness of Kruskal's algorithm by assuming that all edges had distinct weights. Can you modify the proof to prove that if the edge weights are distinct, the Minimum Spanning Tree of the input graph is *unique*?
4. **Binary Trees:** Let T denote a proper binary tree of height h having n nodes. Formally establish that the number of internal nodes in T is at least h and at most $2^h - 1$.
5. **Sorting and Selection:** Given and unordered array $\mathbf{A}$ of n elements, describe how you would find both the maximum and minimum elements of $\mathbf{A}$ using at most $\frac{3}{2}n - 2$ element to element comparisons. You may assume that n is an even number.